# **Magnetic circuit**

#### **<u>1-1 Introduction:</u>**

The electrical machine can be classified according to the motion systems into fixing machines and rotating machines. The transformer is an example for the fixing machines while motor and generator are examples for the rotating machines. The motors can be classified according to the feeding power into D. C. feeding power or A. C. feeding power and the generators can be classified according to the output power into D. C. output power or A. C. output power. The principle operations of these machines are depending upon energy conversions. The forms of energy conversion are different from machine to another where it found that, in the transformer, the electrical energy changes from level to another, in the motor, the electrical energy changes into mechanical energy and in the generator, the mechanical energy changes into electrical energy. All these energy conversions in the electrical machine need medium to pass through it. This medium is a magnetic field so operation of all electrical machines is built on magnetic science. And hence the science of electrical machinery is starting by studying the electromagnetism. In this chapter, the electromagnetism will study. The electromagnetism is studying through explaining the magnetic materials, the definitions of electromagnetism, some important rules in this field and finally comparing between the magnetic circuit and electrical circuit.

# **<u>1-2 Types of Magnetic Materials:</u>**

The materials can be classified according to magnetic into paramagnetic and diamagnetic. This chapter is importance to the paramagnetic materials and effects. The paramagnetic material can be classified into hard magnetic materials and soft magnetic materials.

The hard-magnetic material is called the permanent magnet. it is defined as materials which retain their magnetism and are difficult to demagnetize are called hard magnetic materials. These materials retain their magnetism even after the removal of the applied magnetic field. These materials are used for making permanent magnets. In permanent magnets the movement of the domain wall is prevented. They are prepared by heating the magnetic materials to the required temperature and then quenching them. Impurities increase the strength of hard magnetic materials. They have large hysteresis loss due to large hysteresis loop area. Susceptibility and permeability are low. Coercivity and retentivity values are large. Magnetic energy stored is high. They possess high value of BH product. The eddy current loss is high. They are used in motors, loudspeakers, meters, and holding devices. There are many examples of hard magnetic materials as Steel, Alnico, Rare-Earth Alloys, Hard Ferrites or Ceramic magnets, Bonded Magnets and Nanocrystal line hard magnet (Nd-Fe-B Alloys). Alnico: It is made up of aluminum, nickel and cobalt to boost to improve the magnetic properties. Alnico 5 is the most important material used to create permanent magnet. The BH product is 36000 Jm<sup>-3</sup>. Hard magnetic materials have wide range of applications. They are in automotive as motor drives for fans, wipers, injection pumps; starter motors; Control for seats, windows etc.

The soft-magnetic materials are defined as Soft magnetic materials are easy to magnetize and demagnetize. These materials are used for making temporary magnets. The domain wall movement is easy. They are easy to magnetize. By annealing the cold worked material, the dislocation density is reduced and the domain wall movement is made easier. Soft magnetic materials should not possess any void and its structure should be homogeneous so that the materials are not affected by impurities. They have low hysteresis loss due to small hysteresis area. Susceptibility and permeability are high. Coercivity and retentivity values are less. They have low retentivity and coercivity, they are not used for making permanent magnets. Magnetic energy stored is less. The eddy current loss is less because of high resistivity. There are many examples of soft magnetic materials as iron-silicon alloys, pure iron, nickel iron alloys and ceramic magnet. They are used for manufacturing the cores of transformer and the rotating machines. Adding silicon increase the permeability, decrease the eddy current loss and decrease hysteresis loss this is because these alloys have high resistivity.

# **1-3 Magnetic Circuit:**

A magnetic Circuit in the simplest form can be seen in Fig. 1-1. It consists of core which made from magnetic material and around it there is wound coil. When this coil carries current, a flux ( $\phi$ ) will generate and passes through the core. The strength of this flux depends upon the product of number of turns of the coil (N) and the current passing through the coil (i).



a) Opening magnetic circuit b) closing magnetic circuit Fig. 1-1 the magnetic circuit in simplest form

### **<u>1-4 Important definitions in magnetism:</u>**

There are some important definitions in magnetism must be studied these definitions are Magnetic field:

Generally, the magnetic field can be defined as the portion of the space nearer a magnet body within which the magnetic forces can be detected. The magnetic field can be created by using electromagnet or by using permanent magnet. the electromagnet can be made by passing current through the conductor or through solenoid. A magnetic field is generated when electric charge move through space or within an electrical conductor. The effect of the magnetic field can be noticed by deflecting magnetic needle when placed inside the magnetic field or by generating electromotive force when moving electric wire inside the magnetic field. Magnetic field is also represented by lines of force as static electric field. These lines of force are referred as magnetic flux. They are close together where the field is strong and farther apart where the field is weak. A compass indicates north in the direction of the flux lines. When a unit magnetic pole is placed inside a magnetic field, it will experience both repulsive and attractive force, from similar and opposite poles of the magnet, respectively. The unit pole travels due to resultant of the repulsive and attractive force. The path through which the unit pole travels in the magnetic field is referred as magnetic lines of force.

Force magnetic line:

The magnetic lines of force aren't real. They are imaginary lines of force which draw using north pole i.e. they are drawing from north pole to south pole. Magnetic lines of force are used to represent the strength and orientation of a magnetic field.

The magnetic flux:

The magnetic flux is defined as the number magnetic field lines passing through a closed surface. The symbol of the flux is  $\phi$  and measuring unit is weber. These flux lines have some specific properties that are described below.

- 1. They always form complete closed loops. Unlike lines of electric flux, which radiate from and terminate at the charged surfaces, lines of magnetic flux exist all the way through the magnet.
- 2. They behave as if they are elastic. That is, when distorted they try to return to their natural shape and spacing.
- 3. The lines of force of magnetic field are radiated from the north (N) pole to the south (S) pole.
- 4. Flux lines do not cross or interact to each other.

Flux density:

The flux density can be defined as the flux line per unit area. The symbol of the flux density is B and measuring unit is tesla or weber per meter<sup>2</sup>. the flux density can be calculated from the following rule

$$B = \frac{\phi}{A} \tag{1.1}$$

Ampere law:

The line integral of the magnetic field intensity around a closed path is equal to the sum of the currents flowing through the surface bounded by the path. This can be written as

$$\oint H dl = \sum I$$

Where H is field strength in hennery, dl is pass length in meter and I is the current in ampere.

The source of magnetic field is the ampere turn (N i) this comes from magneto motive force which equal to the ampere law. By applying this into

magnetic circuit it found that;

$$F = Hl = Ni$$

(1.3)

(1.2)

Magnetic field intensity:

Magnetic field intensity at any point within a magnetic field is numerically equally to the force experienced by a *N*-pole of one weber placed at that point or by the ratio between the magnetic flux density to permeability. Permeability is a measure of how easily a magnetic field can set up in a material. permeability takes into account the distribution of the flux within the material. Permeability is the ratio of the flux density of the magnetic field within the material to its field strength. The field intensity is calculated from the following rule

$$H = \frac{B}{\mu} \tag{1.4}$$

Where  $\mu$  is the permeability.

This relation is can be represented graphically as shown in Fig. 2-1. It is called B-H curve. The permeability of any medium or material is

$$\mu = \mu_0 \mu_r \tag{1.5}$$

Where  $\mu_0$  is the vacuum permeability which has constant value 4  $\Pi \ge 10^{-7}$  and  $\mu_0$  is the relative permeability which differ from material to another. The permeability for any material depends upon humidity, temperature, position in the medium and frequency of the applied field.

#### **1-5 Drawing the Hysteresis Loop Curve for any Magnetic Material Practically:**

Hysteresis loop is a four quadrant B - H graph from where the hysteresis loss, coercive force and retentively of s magnetic material are obtained. To understand hysteresis loop, we suppose to take a magnetic material to use as a core around which insulated wire is wound. The coils is connected to the supply (DC) through variable resistor to vary the current I. The magnetic flux density of this core is B which is directly proportional to magnetizing force H. The circuit used to draw the practical B-H curve can be seen in Fig. 1.2



Fig. 1-2 The circuit used to draw the practical B-H curve

Now, let us proceed step by step to make a clear idea about hysteresis loop.

Step 1: When supply current I = 0, so no existence of flux density (B) and magnetizing force (H). The corresponding point is 'O' in the graph above.

Step 2: When current is increased from zero value to a certain value, magnetizing force (H) and flux density (B) both are set up and increased following the path o - a.

Step 3: For a certain value of current, flux density (B) becomes maximum ( $B_{max}$ ). The point indicates the magnetic saturation or maximum flux density of this core material. All element of core material get aligned perfectly. Hence  $H_{max}$  is marked on H axis. So no change of value of B with further increment of H occurs beyond point 'a'.

Step 4: When the value of current is decreased from its value of magnetic flux saturation, H is decreased along with decrement of B not following the previous path rather following the curve a - b.

Step 5: The point 'b' indicates H = 0 for I = 0 with a certain value of B. This lagging of B behind H is called hysteresis. The point 'b' explains that after removing of magnetizing force (H), magnetism property with little value remains in this magnetic material and it is known as residual magnetism (B<sub>r</sub>). Here o - b is the value of residual flux density due to retentivity of the material.

Step 6: If the direction of the current I is reversed, the direction of H also gets reversed. The increment of H in reverse direction following path b - c decreases the value of residual magnetism (B<sub>r</sub>) that gets zero at point 'c' with certain negative value of H. This negative value of H is called coercive force (H<sub>c</sub>)

Step 7: H is increased more in negative direction further; B gets reverses following path c - d. At point 'd', again magnetic saturation takes place but in opposite direction with respect to previous case. At point 'd', B and H get maximum values in reverse direction, i.e. (-B<sub>m</sub> and -H<sub>m</sub>).

Step 8: If we decrease the value of H in this direction, again B decreases following the path de. At point 'e', H gets zero valued but B is with finite value. The point 'e' stands for residual magnetism  $(-B_r)$  of the magnetic core material in opposite direction with respect to previous case.

Step 9: If the direction of H again reversed by reversing the current I, then residual magnetism or residual flux density  $(-B_r)$  again decreases and gets zero at point 'f' following the path e - f. Again, further increment of H, the value of B increases from zero to its maximum value or saturation level at point a following path f - a.

The path a - b - c - d - e - f - a forms hysteresis loop and this can be seen in Fig. 1-3. This path depends upon the shape and the size of the hysteresis loop depend on the nature of the material chosen



Fig. 1-3 B-H curve for magnetic material

From this curve (B-H curve) some important terms can be defined as

Hysteresis of a magnetic material:

It is a property by virtue of which the flux density (B) of this material lags behind the magnetizing force (H).

Coercive force:

It is defined as the negative value of magnetizing force (-H) that reduces residual flux density of a material to zero.

Residual flux density:

It is the certain value of magnetic flux per unit area that remains in the magnetic material without presence of magnetizing force (i.e. H = 0).

#### Retentivity:

It is defined as the degree to which a magnetic material gains its magnetism after magnetizing force (H) is reduced to zero.

Importance of Hysteresis Loop:

The main advantages of hysteresis loop are given below.

1. Smaller hysteresis loop area symbolizes less hysteresis loss.

- 2. Hysteresis loop provides the value of retentivity and coercivity of a material. Thus the way to choose perfect material to make permanent magnet, core of machines becomes easier.
- 3. From B H graph, residual magnetism can be determined and thus choosing of material for electromagnets is easy.

# **<u>1-6 Reluctance of Magnetic Circuit:</u>**

When comparing between the electric circuit and the magnetic circuit it found that,

- 1. The magnetic flux for magnetic circuit is corresponding to the current in the electric circuit.
- 2. Also, the magneto motive force for magnetic circuit is corresponding to the electro motive force in the electric circuit.
- 3. Also, the reluctance for magnetic circuit which resist the passing flux is corresponding to the resistance in the electric circuit.

The last parameter in the magnetic circuit (reluctance) up till now doesn't calculate with using ohm's law and with similarity between the electric circuit and magnetic circuit this parameter can be calculated as

(1.6)

By substituting from eq. 1.1 into equation 1.4 it is found that;  $\phi = \mu A H$ 

By substituting from eq. 1.3 into eq. 1.6 it is found that

$$\frac{Ni}{\phi} = \frac{l}{\mu A} \tag{1.7}$$

By comparing between the right-hand side by similar terms in ohm's low it is found that; this term is similar to the resistance in electric circuit but here it is called reluctance and has symbol  $(\Re)$  so the eq. 1.7 becomes

$$\Re = \frac{l}{\mu A} \tag{1.8}$$

By substituting from eq. 1.5 into eq. 1.8, the final form of reluctance becomes

$$\Re = \frac{l}{(1.9)}$$

$$\mu_{_0}\mu_{_r}A$$

### **1-7 Analysis of Series Magnetic Circuit:**

Simple series magnetic circuit can be seen in Fig. 1-4. The overview of this circuit is at left hand side while the equivalent circuit in the right-hand side. This circuit doesn't contain any airgap. It is uniform core with cross sectional area A, a coil is turning around the core, it has N turns and the current is passing through it, the current has value i. Due to passes current in the coil, the magneto motive force will generate Ni = Hl. Due to effect of the magneto motive force the flux  $\phi$  passes through the core of the magnetic circuit. The direction of the flux can be known by using Fleming hand rule and the mean path of the flux through the core is  $l_m$ . When this flux passes through the core, it resisted by reluctance  $\Re$  of the magnetic core.

The above magnetic circuit doesn't contain any airgap but in rotating machine there is airgap between the magnetic circuit of the stator and magnetic circuit of the rotor so the airgap must be inserted in the studying the magnetic circuit as shown in Fig. 1-5. This Fig. analysis the magnetic circuit with airgap. In this circuit, when applying the current i through the coil which has N number of turns the flux will pass. This flux passes

through the core of the magnetic circuit which has length  $l_m$  and through the airgap which has length  $l_g$  so this magnetic circuit at left hand side in Fig. 1-5 can be represented as the shape Fig. 1-5 at right hand side. So, this circuit has two reluctances one is for the core of magnetic circuit  $\Re_m$  and the other is  $\Re_g$  airgap these reluctances are connected in series. To solve any problems for this circuit uses the following relations depending up on the given and required.



Fig. 1-5 series magnetic circuit with airgap

Due to passing the current through the coil the magneto motive force will generate flux. By applying the ampere law for under this conditions for these circuit it is found that,  $Ni = H_m l_m + H_g l_g$  (1.10)

The field strength for the core of magnetic circuit can be calculated as

$$H_{m} = \frac{B}{\mu_{0}\mu_{r}}$$
(1.11)

The field strength for the airgap of magnetic circuit can be calculated as

$$H_{s} = \frac{B}{\mu_{0}}$$
(1.12)

By substituting from eqs. (1.11) and (1.12) into eq. (1.10) it is found that,

$$Ni = \left(\frac{l_m}{\mu_0 \mu_r} + \frac{l_s}{\mu_0}\right)B \tag{1.13}$$

By substituting about flux density from eq. (1.1) into eq. (1.13) it is found that,

$$Ni = \left(\frac{l_m}{\mu_0 \mu_r^A} + \frac{l_s}{\mu_0^A}\right)\phi \tag{1.14}$$

But the core of the magnetic circuit has reluctance  $\Re_m = \frac{l_m}{\mu_0 \mu_r^A}$  and the airgap has

reluctance  $\Re_{g} = \frac{l_{g}}{\mu_{0}^{A}}$  so eq. (1.14) becomes

$$Ni = (\mathfrak{R}_m + \mathfrak{R}_g)\phi \tag{1.15}$$

Or can be written as

$$\phi = \frac{Ni}{\Re_m + \Re_g} \tag{1.16}$$

### **<u>1-8 Solved examples on the series magnetic circuit:</u>**

### Problem 1-1:

The magnetic circuit for problem 1-1 is shown in Fig. 1-6. This circuit has two sides one of them is thicker if it is compared to the other. The thickness of the core is 20 cm. The relative permeability of the core is 2500. The number of turns for the wound coil is 100 turns. The current flows through the coil is 2 amperes.

- 1- Draw the equivalent circuit.
- 2- Calculate the flux passing through the core.
- 3- Calculate the flux densities in the thicker and thinner sides.
- 4- Calculate the current passing through the coil to produce flux 0.02 weber.



Fig. 1-6 magnetic circuit for example 1-1

Solution:

1- The equivalent circuit



2- Calculating the flux through the magnetic circuit The reluctance for the thicker sides is

$$\Re_{thick er} = \frac{l_{thick er}}{\mu_0 \mu_r A_{thick er}}$$
  

$$\Re_{thinn er} = \frac{l_{thinn er}}{\mu_0 \mu_r A_{thinn er}}$$
  

$$\Re_{thic ker} = \frac{70 \times 10^{-2}}{4\Pi \times 10^{-7} \times 2500 \times (15 \times 20 \times 10^{-4})} = 7424.2424 \quad \text{At/weber}$$
  

$$\Re_{thinn er} = \frac{80 \times 10^{-2}}{4\Pi \times 10^{-7} \times 2500 \times (10 \times 20 \times 10^{-4})} = 12727.27273 \quad \text{At/weber}$$
  

$$\Re_{total} = \Re_{thick er} + \Re_{thinn er}$$
  

$$\Re_{total} = 7424.2424 + 12727.27273 = 20151.51515 \quad \text{At/weber}$$

$$\phi = \frac{Ni}{\Re_{total}}$$

$$\phi = \frac{100x \, 2}{20151.51515} = 0.01 \quad \text{weber}$$

3- Calculating the flux densities in the thicker and thinner sides.

$$B_{thicker} = \frac{\phi}{A_{thicker}}$$

$$B_{thicker} = \frac{0.01}{20x15x10^{-4}} = 0.333 \text{ weber/m}^2$$

$$B_{thinner} = \frac{\phi}{A_{thinner}}$$

$$B_{thinner} = \frac{0.01}{20x10x10^{-4}} = 0.5 \text{ weber/m}^2$$
4- Calculating the current passing through the coil to produce flux 0.02 weber.
$$i = \frac{\Re_{total} \phi}{N}$$

$$i = \frac{20151.51515x \ 0.2}{100} = 4.0303 \text{ ampere}$$
Problem 1-2:

The magnetic circuit for problem 1-2 is shown in Fig. 1-7. There are two coils are wound on the toroid which made from silicon sheet. The first coil carries 0.28 ampere and the other coil carries 0.56 ampere. If the cross-sectional view of this toroid is square find the following with help of B-H curve.

- 1- The flux density in the mean radius of the core.
- 2- The flux passing through the mean radius of the core.
- 3- The relative permeability of the core.



B-H Curve for different materials



Fig. 1-7 magnetic circuit for example 1-2

### **Solution:**

1- The flux density in the mean radius of the core.

From magnetic circuit, it is found that; the two fluxes from the coils are aided and by applying the ampere law

$$mmf = N_{1}i_{1} + N_{2}i_{2}$$

$$mmf = 600x0.28 + 300x \ 0.56 = 336 \ \text{At}$$
The mean pass  $(l_{m}) = 2 \ \Pi \left(\frac{19 + 21}{2}\right) = 40 \ \Pi \ \text{cm}$ 

$$N_{1}i_{1} + N_{2}i_{2} = Hl_{m}$$

$$H = \frac{N_{1}i_{1} + N_{2}i_{2}}{l_{m}}$$

$$H = \frac{336}{40\Pi x \ 10^{-2}} = 267.38 \ \text{At/m}$$

From the B-H curve the flux density for silicon is  $1.1 \text{ weber/m}^2$ 

2- The flux passing through the mean radius of the core.

 $\phi = 1.1x (2x 2x 10^{-4}) = 4.4x 10^{-4}$  weber

3- The relative permeability of the core.

$$\mu_{r} = \frac{B}{\mu_{0}H}$$

$$\mu_{r} = \frac{1.1}{4\Pi x \, 10^{-7} x \, 267.38} = 3273.82$$
Problem 1-3:

<u>Problem 1-3:</u>

Fig. 1-8 shows synchronous machine with the following physical parameters airgap length is 1cm, the applied voltage through the rotor coil is 100 voltages and the total resistance of the rotor coil is 10 Ohm. The number of turns for this coil is 1000 turns, the rotor face pole area is  $0.2 \text{ m}^2$  assume that the rotor and stator flux of synchronous machine have negligible reluctance (infinite permeability) and neglecting the fringing find the following

- 1- Draw the magnetic circuit.
- 2- Calculate the magneto motive force.
- 3- Calculate the reluctance of each airgap.
- 4- Calculate the total magnetic flux in each airgap.
- 5- Calculate the magnetic flux density in each airgap.



Fig. 1-8 synchronous machine

#### **Solution:**

1- Drawing the magnetic circuit.



2- Calculate the magneto motive force.

The rotor current is  $i = \frac{v}{R} = \frac{100}{10} = 10$  Amperes Ni = 1000x 10 = 10000 Ampere turn 3- Calculating the reluctance of each airgap.  $\Re_{airgap} = \frac{l_s}{\mu_0^A}$   $\Re_{airgap} = \frac{0.01}{4\Pi x 10^{-7} x 0.2} = 39788.735$  At/weber 4- Calculating the total magnetic flux in each airgap.  $\phi = \frac{Ni}{2x \Re_{airgap}}$   $\phi = \frac{10000}{2X 39788.735} = 0.12566$  weber 5- Calculate the magnetic flux density in each airgap.  $B = \frac{\phi}{A}$  $B = \frac{0.12566}{0.2} = 0.6283$  weber/m<sup>2</sup>

# **1-9 Leakage Flux & Fringing Effect:**

In the magnetic circuit of Fig. 1-9 an air gap is present. For an exciting current, the flux lines produced are shown. These flux lines cross the air gap from the top surface of the core to the bottom surface of the core. So, the upper surface behaves like a north pole and the bottom surface like a south pole. Thus, all the flux lines will not be vertical and confined to the core face area alone. Some lines of force in fact will reach the bottom surface via bulged out curved paths outside the face area of the core. These fluxes which follow these curved paths are called fringing flux and the phenomenon is called fringing effect. Obviously, the effect of fringing will be smaller if the air gap is quite small. Effect of fringing is to make flux density in the air gap a bit less than in the core as in the air same amount of flux is spread over an area which is greater than the core sectional area. Unless otherwise specified, we shall neglect the fringing effect in our following discussion. Effect of fringing sometimes taken into account by considering the effective area in air to be about 10 to 12% higher than the core area.



Fig. 1-9 Fringing effect due to airgap 1-10 Solved examples on the series magnetic circuit:

#### Problem 1-4:

A two-legged magnetic core with an air gap and magnetization curve of the core material are shown in Fig.1-10. The depth of the core is 5 cm, the length of the air gap in the core is 0.07 cm, and the number of turns on the coil is 500. Assume a 5 percent increase in effective air-gap area to account for fringing. Find the following

- 1. The current is required to produce an air-gap flux density of 0.5 T.
- 2. The flux densities of the four sides of the core at that current.
- 3. The total flux present in the air gap.





### Solution:

The flux through the airgap can be calculated as

 $\phi_{g} = B_{g}A_{geffect}$  $\phi_{g} = 0.5x \ 0.05x \ 0.05x \ 1.05 = 1.3125x \ 10^{-3}$  weber

Due to the flux through the airgap = the flux through the core i.e.  $\phi_{g} = \phi_{core}$  so flux density through the limb of the right-hand side is

$$B_{ight \ limb} = \frac{\phi_{core}}{A_{ight \ limb}}$$
$$B_{ight \ limb} = \frac{1.3125 \times 10^{-3}}{0.05 \times 0.05} = 0.525 \quad \text{Tesla}$$
$$B_{lift \ limb} = B_{top \ limb} = B_{bottom \ limb} = \frac{1.3125 \times 10^{-3}}{0.1 \times 0.05} = 0.2625 \quad \text{Tesla}$$

The field intensity in the airgap can be found from flowing law

$$H_{g} = \frac{B_{g}}{\mu_{0}}$$
  
 $H_{g} = \frac{0.5}{4\Pi x \, 10^{-7}} = 397887.3 \text{ At/m}$ 

The field intensity in the right limb can be found from the magnetization curve as At  $B_{right \ limb} = 0.525$  Tesla, the field intensity  $H_g = 410$  At/m

The field intensity in the left limb, top limb, bottom limb can be found from the magnetization curve as

At 
$$B_{lift limb} = B_{top limb} = B_{bottom limb} = 0.2625$$
 Tesla, the field intensity of  
 $H_{lift limb} = H_{top limb} = H_{bottom limb} = 240$  At/m  
The total mmf for the magnetic circuit is  
 $mmf = H_g l_g + H_{right} limb l_{right limb} + H_{lift limb} l_{lift limb} + H_{top limb} l_{top limb} + H_{bottom limb} l_{bottom limb}$   
 $mmf = 397887.3x \ 0.0007 + 410x \ 0.4 + 240(0.4 + 0.4 + 0.4) = 731$  At  
 $i = \frac{mmf}{N}$   
 $i = \frac{731}{500} = 1.462$  ampere  
The flux densities in the four sides of the core and the total flux present

The flux densities in the four sides of the core and the total flux present in the air gap were calculated above.

## **1-11 Analysis of Series and Parallel Magnetic Circuit:**

In Fig. 1-11, PU, QT and RS are the limbs whereas PQ, QR, UT and TS are the yokes. It is customary to fix up the corner points P,Q,R etc. from the given physical dimensions, joining of which will give you the *mean length* of the flux paths. If the coil carries a current i in the direction shown, flux  $\phi$ , produced in the first limb will be in the upward direction. Same  $\phi$  is constrained to move along the yoke PQ. At point Q, two parallel paths are available to  $\phi$  for its onwards journey namely (i) the central limb QT and (ii) the yoke QR. In other words,  $\phi$  will be divided into two components  $\phi_1$  and

 $\phi_2$  as shown with the obvious condition  $\phi = \phi_1 + \phi_2$ . The relative values of these components will be decided by respective reluctances of the paths.  $\phi_1$  and  $\phi_2$  once again recombine at point T and completes the path. Now in the path TUPQ flux  $\phi$  is same, it is made of same material and has same cross sectional area A, then its reluctance  $\Re_{TUPQ} \alpha \frac{l_{TUPQ}}{A}$ . In the central limb, flux is same ( $\phi_1$ ), however it encounters two materials, one is iron (QM and WT) and the other is a small air gap (MW). The reluctance of the air gap  $\Re_g = \frac{l_g}{\mu_0^A}$ . The two reluctances  $\Re_{QM}$  and  $\Re_{WT}$  of the magnetic material may however be combined into a single reluctance as  $\Re_1 = \Re_{OM} + \Re_{WT}$ . The portion of the magnetic

circuit which carries flux  $\phi_2$  can be represented by a single reluctance  $\Re_{QR ST} \alpha \frac{l_{QR ST}}{A}$ . Instead of carrying on with long suffixes let us call  $\Re_{QRST}$  to be  $\Re_2$ . To write down the basic equations let us redraw the electrical equivalence Fig.1-11 to Fig. 1-12.



Fig. 1-11 Series and Parallel Magnetic Circuit



Fig. 1-12 redraw the electrical equivalence Fig.1-11 **<u>1-12 Solved examples on the series parallel magnetic circuit:</u>** 

#### Problem 1-5:

A core with three legs is shown in Fig. 1-13. Its depth is 5 cm, and there are 200 turns on the left most leg. The relative permeability of the core can be assumed to be 1500 and constant. What flux exists in each of the three legs of the core? What is the flux density in each of the legs? Assume a 4% increase in the effective area of the air gap due to fringing effects.



Fig. 1-13 Magnetic circuit for problem 1-5

Solution:

This core can be divided up into four regions. Let  $\mathfrak{R}_1$  be the reluctance of the left-hand portion of the core,  $\mathfrak{R}_2$  be the reluctance of the center leg of the core,  $\mathfrak{R}_3$  be the reluctance of the center air gap, and  $\mathfrak{R}_4$  be the reluctance of the right-hand portion of the core. Then the total reluctance of the core is

$$\Re_{TOT} = \Re_1 + \frac{(\Re_2 + \Re_3)\Re_4}{\Re_2 + \Re_3 + \Re_4}$$
  
$$\Re_1 = \frac{l_1}{\mu_0 \mu_r A_1} = \frac{1.08}{4\Pi x \, 10^{-7} x \, 1500x \, (0.09x \, 0.05)} = 127324 \text{ At/weber}$$

$$\Re_{2} = \frac{l_{2}}{\mu_{0}\mu_{r}A_{2}} = \frac{0.34}{4\Pi x \, 10^{-7} \, x \, 1500x \, (0.15x \, 0.05)} = 24050 \text{ At/weber}$$
$$\Re_{3} = \frac{l_{3}}{\mu_{0}A_{3}} = \frac{0.0004}{4\Pi x \, 10^{-7} \, x \, (0.15x \, 0.05)x \, 1.04} = 40809 \text{ At/weber}$$
$$\Re_{4} = \frac{l_{4}}{\mu_{0}\mu_{r}A_{4}} = \frac{1.08}{4\Pi x \, 10^{-7} \, x \, 1500x \, (0.09x \, 0.05)} = 127324 \text{ At/weber}$$

The total reluctance is

$$\Re_{TOT} = \Re_1 + \frac{(\Re_2 + \Re_3)\Re_4}{\Re_2 + \Re_3 + \Re_4} = 127324 + \frac{(24050 + 40809)x127324}{24050 + 40809 + 127324} = 170294 \text{ At/weber}$$

The total flux in the core is equal to the flux in the left leg:

$$\phi_{total} = \phi_{left} = \frac{Ni}{\Re_{total}} = \frac{200x}{170294} = 0.0023488$$
 weber

The fluxes in the center and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{center} = \frac{\Re_4}{\Re_2 + \Re_3 + \Re_4} \phi_{total} = \frac{127324}{24050 + 40809 + 127324} \times 0.0023488 = 0.001555 \text{ At/weber}$$
$$\phi_{right} = \frac{\Re_2 + \Re_3}{\Re_2 + \Re_3 + \Re_4} \phi_{total} = \frac{24050 + 40809}{24050 + 40809 + 127324} \times 0.0023488 = 0.0007926 \text{ At/weber}$$

The flux density in the legs can be determined from the equation  $\phi = BA$ 

$$B_{left} = \frac{\phi_{left}}{A} = \frac{0.0023488}{0.09x \ 0.05} = 0.522 \text{ Tesla}$$
$$B_{center} = \frac{\phi_{center}}{A} = \frac{0.001555}{0.15x \ 0.05} = 0.20733 \text{ Tesla}$$
$$B_{right} = \frac{\phi_{right}}{A} = \frac{0.0007926}{0.09x \ 0.05} = 0.17613 \text{ Tesla}$$

#### **1-13 Induced Voltage and Force on a Conductor Moving in a Magnetic Field:**

If a wire moves through a magnetic field, a voltage is induced in it. This is a simple idea of the electricity generation and this is shown in Fig.1-14 The voltage induced in the wire can be determined as

$$\boldsymbol{e}_{ind} = (vXB) \bullet I \tag{1.17}$$

Where v is the velocity of the wire, B is a magnetic flux density vector and I is the current passing through the wire.



Fig. 1-14 Wire carrying current and moving in magnetic field

# **1-14 Force on a Conductor Carrying Current Placed in a Magnetic Field:**

If a wire carrying current is placed in a magnetic field, a force is induced in it. This is a simple idea of the motor action and this is shown in Fig.1-15 The voltage induced in the wire can be determined as

F = (LXB)I

(1.18)

Where L is the length of the wire, B is a magnetic flux density vector and I is the current passing through the wire.



Fig. 1-15 Wire carrying current and placed in a magnetic field **1-15 Solved examples on the motor and generator actions:** 

# Problem 1-6:

A wire has length 0.25 meter is placed in the magnetic field has flux density 0.5 Tesla find

- 1. The induced voltage on a wire if it moves with velocity 10 m/s perpendicular the magnetic field
- 2. The induced voltage on a wire if it moves with velocity 10 m/s when the smallest positive angle between vXB and the wire is 60°
- 3. If a wire in case one carrying current 5 ampere find the force effected on it
- 4. If the angle between wire and flux density is 30° find the force effect it when carrying current 10 ampere

### Solution:

The induced voltage in the first case is

 $e_{ind} = (vXB) \bullet I = 10X \ 0.5X \ 0.25 = 1.25$  Voltages

The induced voltage in the second case is

 $e_{ind} = (vXB) \bullet I = 10X \ 0.5X \ 0.25X \ \cos 60^{\circ} = 0.625$  Voltages

The induced force in the third case is

 $F = (LXB)I = 0.25X \ 0.5X \ 5 = 0.625$  Newton

The induced voltage in the fourth case is

 $F = (LXB)I = 0.25X \ 0.5X \ 10X \ \sin 30^{\circ} = 0.625$  Newton

# Year work 1 sheet 1

# Problem 1:

A ferromagnetic core is shown in the following Fig. Three sides of this core are of uniform width. while the fourth side is somewhat thinner. The depth of the core (into the page) is 10 cm. and the other dimensions are shown in the figure. There is a 200 turn coil wrapped around the left side of the core. Assuming relative permeability is 2500. how much flux will be produced by a I-A input current?



# Problem 2:

The following Fig. shows a ferromagnetic core whose mean path length is 40 cm. There is a small gap of 0.05 cm in the structure of the otherwise whole core. The cross-sectional area of the core is  $12 \text{ cm}^2$ , the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assuming that fringing in the air gap increases the effective cross-sectional area of the air gap by 5 percent. Given this information, Find:

(a) the total reluctance of the flux path (iron plus air gap)

(b) the current required to produce a flux density of 0.5 T in the air gap.



#### Problem 3:

A ferromagnetic core with a relative permeability of 1500 is shown in the following Fig. The dimensions are as shown in the diagram. and the depth of the core is 7 cm. The air gaps on the left and right sides of the core are 0.070 and 0.050 cm. respectively. Because of fringing effects. the effective area of the air gaps is 5 percent larger than their physical size. If there are 400 IIU11S in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A. what is the flux in each of the left. center. and right legs of the core? What is the flux density in each air gap?



# Problem 4:

A two-legged core is shown in the following Fig. The winding on the left leg of the core  $(N_1)$  has 400 turns. and the winding on the right  $(N_2)$  has 300 turns. The coils are wound in the directions shown in the Fig. If the dimensions are as shown. Then what flux would be produced by currents  $i_1 = 0.5$  A and  $i_2 = 0.75$  A? Assume relative permeability is 1000.



# **DIRECT CURRENT MACHINES**

#### 2-1 Introduction:

Direct current machines are used in many industrial applications. It is used as the generator and as the motor. The generator is used to convert the mechanical energy into electrical energy and the motor is used to convert the electrical energy into mechanical energy. The operation of these machines depends upon the Faraday experiment.

In this chapter, the following will explain

- 1. Theory of operation for DC machine.
- 2. Construction of DC machine.
- 3. Commutator winding of DC machine.
- 4. Armature reaction problems and solution in DC machine.
- 5. The induced voltage and the induced torque in real DC machine.

# **2-2 Theory of Operation for DC Generator:**

To understand the operation of DC generator, simple DC machine is used. It is in Fig. 2-1. It consists of a simple loop of wire rotating around fixed axis. The rotating loop is called a rotor while the stationery part is called a stator. The magnetic field for this DC machine is supplied by north and south pole in Fig. 2-1. When the rotor is rotating in the magnetic field, the induced voltage will generate in the wire loop. The generating voltage inside the wire loop depends up on Faraday law. This voltage can be calculated from the following relation

$$\boldsymbol{\varrho}_{ind} = (\boldsymbol{v} \ \boldsymbol{x} \ \boldsymbol{B}) \bullet \boldsymbol{l} \tag{2.1}$$

Where v is linear cutting velocity for the flux in meter, B is the flux density in tesla and l the conductor length in meter



Fig. 2-1 Electrical machines operation idea

By examining Fig. 2-1, it is found that, the two sides ab and cd are perpendicular the magnetic field where the two sides bc and ad are parallel to the magnetic field. By applying the Faraday law and Flaming rule (Fig. 2-1-b) on the rotating loop abcd (Fig. 2-1-a) it is found that,

Generating induced voltage  $e_{ind}$  in the segment ab and the direction of this voltage is into

page. The same voltage generates in segment cd and the direction of this voltage is out page. The inducing voltage is maximum value because the magnetic field is perpendicular the two segments ab and cd. At this time the inducing voltages in the two segments be and ad are zero this because these segments parallel the magnetic field. So, the total inducing voltage are 2vBl. When the rotating loop abcd rotates the inducing voltage in the two segments cd and da increase because the inducing voltage depends upon the sine angle between the direction of the movement and magnetic field. When the two segments ab and cd become parallel to

the magnetic field, the inducing voltages become zero volt while the two segments cd and da become perpendicular to the magnetic field, the inducing voltages become 2vBl volt. With rotation continuity for the wiring loop, the inducing voltage appear another time but in the opposite direction. When these segments become perpendicular the magnetic field another time where the segment ab and the direction of this voltage is out page. The same voltage generates in segment cd and the direction of this voltage is into page. the inducing voltages becomes maximum value in the opposite direction i.e. the angle between the motion of these segments become  $270^{\circ}$ . At this time the inducing voltages in the two segments bc and ad are zero this because these segments parallel the magnetic field. So, the total inducing voltage are - 2vBl . after this position, the inducing voltages decrease in the negative direction up to reach zero voltage. With rotation continuity, the inducing voltage rebate. The shape of the inducing voltages for one cycle (one rotating) can be seen in Fig. 2-1-c. The shape of inducing voltages is sine wave to get inducing voltage has constant magnitude and direction with varying time, this voltage must be rectified before out of rotating loop. This occurs by replacing the two sliprings by two circular conducting segment A, B as in Fig. 2-2a. The two fixed contacts are setup at an angle such that at the instant when the voltage in the loop is zero, the contacts short-circuit the two segments. In this fashion, every time the voltage of the loop switches direction, the contacts also switch connections, and the output of the contacts is always built up in the same way in Fig.2-2b. This connection-switching process is known as commutation. The rotating semicircular segments are called commutator segments, and the fixed contacts are called brushes. By this way, the direction of the induced voltage becomes constant but the intensity of the voltage doesn't constant to get rid the problem more coils are used and this will illustrate latter.



Fig. 2-2 The DC machine and output voltage after commutating

# **<u>2-3 Construction of the DC machine:</u>**

The DC machine consists of two main part as shown in Fig. (2-3), these parts are stator and rotor. The stator makes magnetic field where the rotor is called armature and generated in

it the induced electromotive force i.e. the mechanical energy converts into electrical energy and the airgap separating between stator and rotor.

2-3-1 The stator:

The stator consists of yoke, main pole, field coils and commutating poles. The stator parts explain as,

2-3-1a Yoke:

Fig. 2-4 shows the yoke. It made from wrought iron, cast iron or steel chips for small machine. It is used as pass to complete the magnetic circuit and the poles fixed in it. 2-3-1b Main poles:

Main poles made from steel chips. It is fixed in the yoke and pole shoe is constructed on it. The pole shoe is used for distributing and regularity of flux lines in the airgap.

2-3-1c Field coils:

Field coils are two types of field coils, shunt field coils and series field coils. These coils (shunt field coils and series field coils) generate the magnetic field when the current passing through them. These coils warped around the main pole and note around the pole shoe. These field coils make from insulated copper wire or from slices insulated cooper. This can be seen in Fig. 2-4 where this Fig. shows the main pole, pole shoe and field coils.



Fig. 2-2 DC machine scheme



Fig. 2-3 The yoke for DC machine



Fig. 2-4 The main pole and field coils for DC machine

2-3-1d Commutating poles:

Commutating poles are similar to the main poles but they are smaller. They are inserted among the main poles. They are fixed on yoke. They are commutating coils around the commutating poles to improve the problems come from commutating.

# 2-3-2 The rotor:

The rotor consists of armature core, armature winding, commutator and brush holder.

2-3-2a Armature core:

It is shown in Fig. 2-5. It is a cylindrical part. It is made from electrically isolated compressed steel chips. It is isolated by varnish to decrease the eddy current. There are slots on external peripheral of the armature core to setup the armature coils inside it. Also, there is fan to Colling the DC machine constructed on the rotor shaft.



Fig. 2-5 The armature core

### 2-3-2b Armature windings:

The armature core is provided with slots made of the same material as the core to which the armature winding made with several turns of copper wire distributed uniformly over the entire periphery of the core. The slot openings a shut with fibrous wedges to prevent the conductor from plying out due to the high centrifugal force produced during the rotation of the armature, in presence of supply current and field.

### 2-3-2c Commutators:

Commutators is a moving part of a rotary electrical switch in certain types of electric motors and electrical generators that periodically reverses the current direction between the rotor and the external circuit. It consists of a cylinder composed of multiple metal contact segments on the rotating armature of the machine. Two or more electrical contacts called "brushes" made of a soft conductive material like carbon press against the commutator, making sliding contact with successive segments of the commutator as it rotates. The windings (coils of wire) on the armature are connected to the commutator segments.

### 2-3-2d Brush holder:

Brush holder is a spring is typically used with the brush, to maintain constant contact with the commutator this brush holder can be seen in Fig. 2-6. As the brush and commutator wear down, the spring steadily pushes the brush downwards towards the commutator. Eventually the brush wears small and thin enough that steady contact is no longer possible or it is no longer securely held in the brush holder, and so the brush must be replaced. It is common for a flexible power cable to be directly attached to the brush, because current flowing through the support spring would cause heating, which may lead to a loss of metal temper and a loss of the spring tension.

### 2-3-2e Brush and contact angle:

brush and location on the commutator can be seen in Fig. 2-7 forms an important component in the process of commutation. The coil resistance is normally very small compared to the brush contact resistance. Further this brush contact resistance is not a constant. With the brushes commonly used, an increase in the current density of the brushes by 100 percent increases the brush drop by about 10 to 15 percent. Brush contact drop is influenced by the major factors like speed of operation, pressure on the brushes, and to a smaller extent the direction of current flow.

Major change in contact resistance is brought about by the composition of the brush. Soft graphite brushes working at a current density of about 10A/cm2 produce a drop of 1.6V (at the positive and negative brushes put together) while copper-carbon brush working at 15A/cm2 produces a drop of about 0.3V. The coefficient of friction for these brushes are 0.12 and 0.16 respectively. The attention is focused next on the process of commutation. The different brush types make contact with the commutator in different ways. Because copper brushes have the same hardness as the commutator segments, the rotor cannot be spun backwards against the ends of copper brushes without the copper digging into the segments and causing severe damage. Consequently, strip/laminate copper brushes only make tangential contact with the commutator, while copper mesh and wire brushes use an inclined contact angle touching their edge across the segments of a commutator that can spin in only one direction. The softness of carbon brushes permits direct radial end-contact with the commutator without damage to the segments, permitting easy reversal of rotor

direction, without the need to reorient the brush holders for operation in the opposite direction. Although never reversed, common appliance motors that use wound rotors, commutators and brushes have radial-contact brushes. In the case of a reaction-type carbon brush holder, carbon brushes may be reversely inclined with the commutator so that the commutator tends to push against the carbon for firm contact.



Fig. 2-6 The brush holder



Fig. 2-7 The brush location

# **2-4 Winding Types:**

Armature windings are connecting in series or in parallel. They are connecting in series for DC machine which carrying high current or connecting in parallel for DC machine which operating at high voltage. The terminal windings of the armature windings are welded with commutating segment. The armature windings are mainly having two types of the windings. These two types are lap winding and wave winding. Here we are going to discuss about them.

# 2-4-1 Lap winding:

Lap winding is the winding in which successive coils overlap each other. It is named "Lap" winding because it doubles or laps back with its succeeding coils. In this winding the finishing end of one coil is connected to one commutator segment and the starting end of the next coil situated under the same pole and connected with same commutator segment. In Fig. 2-8, the finishing end of coil - 1 and starting end of coil - 2 are both connected to the commutator segment – 2. These coils are under the same magnetic pole that is N pole. lap winding has two types simplex lap winding and duplex lap Winding.

# 2-4-1a Simplex lap:

A winding in which the number of parallel path between the brushes is equal to the number of poles is called simplex lap winding.

# 2-4-1b duplex lap winding:

A winding in which the number of parallel path between the brushes is twice the number of poles is called duplex lap winding.



Fig. 2-8 The lap winding

2-4-1c Advantages of the lap winding:

- 1. This winding is necessarily required for large current application because it has more parallel paths.
- 2. It is suitable for low voltage and high current generators.
- 2-4-1d Disadvantages of the lap winding:
- 1. It gives less emf compared to wave winding. This winding requires more no. of conductors for giving the same emf, it results high winding cost.
- 2. It has less efficient utilization of space in the armature slots.

# 2-4-2 Wave winding:

Wave winding is one type of armature winding. It is shown in Fig. 2-9. In this winding the end of one coil is connected to the starting of another coil of the same polarity as that of the first coil. In this type of winding the coil side(A-B) progress forward around the armature to another coil side and goes on successively passing through N and S pole till it returns to a conductor (A1-B1) lying under the starting pole. This winding forms a wave with its coil, that's why it is named as wave winding. It is also called series winding because its coils are connected in series.

2-4-2a Progressive wave winding:

If after one round of the armature the coil falls in a slot right to its starting slot the winging is called Progressive wave winding.

2-4-2b Reprogressive wave winding:

If after one round of the armature the coil falls in a slot left to its starting slot the winging is called Retrogressive wave winding.

2-4-2c Advantages of the wave winding:

- 1. In this winding, only two brushes are required but more parallel brushes can be added to make it equal to the no. of poles. If one or more brushes set poor contacts with the commutator, satisfactory operation is still possible.
- 2. This winding gives sparkles commutation. The reason behind that it has two parallel paths irrespective of no of poles of the machine. The conductors in each of the two parallel path distributed around the armature in the entire circumference.

- 3. For a given no of poles and armature conductors it gives more emf than that of lap winding. Hence wave winding is used in high voltage and low current machines. This winding is suitable for small generators circuit with voltage rating 500-600V.
- 2-4-2d Disadvantages of the wave winding:
- 1. Wave winding cannot be used in the machines having higher current rating because it has only two parallel paths



Fig. 2-9 The wave winding

# 2-5 Problems with Commutation in Real Machines:

There are two main problems come from commutation in real DC machine. These problems are:

- 1. Armature reaction
- 2. Kick voltages  $L \frac{di}{dt}$
- 2-5-1 Armature reaction:

The armature reaction means an effect of armature flux on the main field flux. This can be explained as, if the magnetic field windings of a DC machine are connected to a power supply and the rotor of the machine is turned by an external source of mechanical power, a voltage will be induced in the conductors of the rotor. This voltage will be rectified into a DC output by the action of the machine's commutator. If connecting a load to the terminals of the machine, and a current will flow in its armature windings. This current flow will produce a magnetic field of its own, which will distort the original magnetic field from the machine's poles. This distortion of the flux in a machine as the load is increased is called armature reaction. It causes two serious problems in real DC machines as,

- 1. Neutral-plane shift.
- 2. Flux weakening.

<u>2-5-1a The magnetic neutral plane:</u>

EMF is induced in the armature conductors when they cut the magnetic field lines. But, there is an axis along or you may say a plane which armature conductors move parallel to the flux lines and, hence, they do not cut the flux lines at the moment. Magnetic Neutral Axis (MNA) may be defined as the axis along which no emf is generated in the armature conductors as they move parallel to the flux lines. Brushes are always placed along MNA because reversal of current in the armature conductors takes place along this axis.

Geometrical Neutral Axis (GNA) may be defined as the axis which is perpendicular to the stator field axis. In ideal case, there isn't shifting between the magnetic neutral axis and geometrical neutral axis but when there is shifting between the magnetic neutral axis and geometrical neutral axis the armature reaction appears. To understand the problem of neutral-plane shift, examine Fig. 2- 10. This Fig. shows a two-pole dc machine. The flux in that machine is distributed uniformly under the pole faces. The rotor windings shown have voltages built up out of the page for wires under the north pole face and into the page for wires under the south pole face. The neutral plane in this machine is exactly vertical.



### Main field flux

Fig. 2-10 Uniformly pole flux when the magnetic neutral plane is vertical Fig. 2-11 shows armature flux lines due to the armature current. In case the machine is running, both the fluxes (flux due to the armature conductors and flux due to the field winding) will be present at a time. The armature flux superimposes with the main field flux and, hence, disturbs the main field flux (as shown in Fig. 2-12 the of above image). This effect is called as armature reaction in DC machines.



Armature Flux

Fig. 2-11 The magnetic field comes from the rotor current

This can more explain as if a load is connected to this machine so that it acts as a generator. This current flow produces a magnetic field from the rotor windings, as shown in Fig. 2-11. This rotor magnetic field affects the original magnetic field from the poles that produced the generator's voltage in the first place. In some places under the pole surfaces, it subtracts from the pole flux, and in other places it adds to the pole flux. The overall result

is that the magnetic flux in the air gap of the machine is skewed as shown in Figs 2- 12. Notice that the place on the rotor where the induced voltage in a conductor would be zero (the neutral plane) has shifted.



Distortion of main field flux due to armature flux - Armature reaction

Fig. 2- 12 The new neutral plane compared to old neutral plane For the generator shown in Fig. 2- 12, the magnetic neutral plane shifted in the direction of rotation. If this machine had been a motor, the current in its rotor would be reversed and the flux would bunch up in the opposite corners from the bunches shown in the Fig. As a result, the magnetic neutral plane would shift the other way. This means that if loaded DC generator, MNA will be shifted in the direction of the rotation. On the other hand, for a loaded dc motor, MNA will be shifted in the direction opposite to that of the rotation. In general, the neutral plane shifts in the direction of motion for a generator and opposite to the direction of motion for a motor.

Furthermore, the amount of the shift depends on the amount of rotor current i.e. on the load of the machine. So, what's the big deal about neutral-plane shift? It's just this: The commutator must short out commutator segments just at the moment when the voltage across them is equal to zero. If the brushes are set to short out conductors in the vertical plane, then the voltage between segments is indeed zero until the machine is loaded. When the machine is loaded, the neutral plane shifts, and the brushes short out commutator segments with a finite voltage across them. The result is a current flow circulating between the shorted segments and large sparks at the brushes when the current path is interrupted as the brush leaves a segment. The end result is arcing and sparking at the brushes. This is a very serious problem, since it leads to drastically reduced brush life, pitting of the commutator segments, and greatly increased maintenance costs. Notice that this problem cannot be fixed even by placing the brushes over the full-load neutral plane, because then they would spark at no load. In extreme cases, the neutral-plane shift can even lead to flashover in the commutator segments near the brushes. The air near the brushes in a machine is normally ionized as a result of the sparking on the brushes. Flashover occurs when the voltage of adjacent commutator segments gets large enough to sustain an arc in the ionized air above them. If flashover occurs, the resulting arc can even melt the commutator's surface.

2-5-1b The second major problem caused by armature reaction:

The second major problem caused by armature reaction is called flux weakening. To understand flux weakening, the magnetization curve must be studied. It is shown in Fig. 2-

13. Most machines operate at flux densities near the saturation point. Therefore, at locations on the pole surfaces where the rotor magnetomotive force adds to the pole magnetomotive force, only a small increase in flux occurs. But at locations on the pole surfaces where the rotor magnetomotive force subtracts from the pole magnetomotive force, there is a larger decrease in flux. The net result is that the total average flux under the entire pole face is decreased as shown in Fig. 2-14. Flux weakening causes problems in both generators and motors. In generators, the effect of flux weakening is simply to reduce the voltage supplied by the generator for any given load. In motors, the effect can be more serious. When the flux in a motor is decreased, its speed increases. But increasing the speed of a motor can increase its load, resulting in more flux weakening. It is possible for some shunt dc motors to reach a runaway condition as a result of flux weakening, where the speed of the motor just keeps increasing until the machine is disconnected from the power line or until it destroys itself.

2-5-2 Kick voltage:

Kick voltage is the  $L \frac{di}{dt}$  voltage that occurs in commutator segments being shorted out by the brushes. To understanding this problem, look at Fig. 2-15. Notice that when a

commutator segment is shorted out, the current flow through that commutator segment must reverse. With even a tiny inductance in the loop, a very significant inductive voltage

kick v =  $L \frac{di}{dt}$  will be induced in the shorted commutator segment. This high voltage

naturally causes sparking at the brushes of the machine, resulting in the same arcing problems that the neutral-plane shift causes.



 $\Delta \phi_{i}$  = flux increase under reinforced sections of poles  $\Delta \phi_d$  = flux decrease under subtracting sections of poles Fig. 2-13 A typical magnetization curve shows the effects of pole saturation where

armature and pole magnetomotive forces add



Fig. 2-14 The flux and magnetomotive force under the pole faces in a DC machine. Assuming that the current in the brush is 400 A, the current in each path is 200 A. Assuming that the machine is turning at 800 r/min and that there are 50 commutator segments (a reasonable number for a typical motor), each commutator segment moves under a brush and clears it again in t = 0.00 15 s. Therefore, the rate of change in current with respect to time in the shorted loop must average



Fig. 2-15 Effect of kick voltage

### 2-6 Solution of the Effect of Armature Reaction:

The effect of armature reaction can be solved by Brush shifting, inter-poles compensating winding.

2-6-1 Brush shifting:

Someone had to adjust the brushes to the new neutral point every time the load on the machine changed. shifting the brushes may have stopped the brush sparking, but it actually aggravated the flux-weakening effect of the armature reaction in the machine.

- 1. This is true because of two effects: The rotor magnetomotive force now has a vector component that opposes the magnetomotive force from the poles this shows in Fig. 2-16.
- 2. The change in armature current distribution causes the flux to bunch up even more at the saturated parts of the pole faces.

Another slightly different approach sometimes taken was to fix the brushes in a compromise position (say, one that caused no sparking at two-thirds of full load). In this case, the motor sparked at no load and somewhat at full load Today, brush shifting is only used in very small machines that always run as motors.



Fig. 2-16 Net magnetomotive force in DC machine



Because of the requirement that a person must adjust the brush positions of machines as their loads change, another solution to the problem of brush sparking was developed. The basic idea behind this new approach is that if the voltage in the wires undergoing commutation can be made zero, then there will be no sparking at the brushes. To accomplish this, small poles, called commutating poles or inter-poles as shown in Fig. 2-17 are placed midway between the main poles. These commutating poles are located directly over the conductors being commutated. By providing a flux from the commutating poles, the voltage in the coils undergoing commutation can be exactly canceled. If the cancellation is exact, then there will be no sparking at the brushes. The commutating poles do not otherwise change the operation of the machine, because they are so small that they affect only the few conductors about to undergo commutation. Notice that the armature reaction under the main pole faces is unaffected, since the effects of the commutating poles do not extend that far. This means that the flux weakening in the machine is unaffected by commutating poles. They are connected in series with the windings on the rotor. As the load increases and the rotor current

increases, the magnitude of the neutral-plane shift and the size of the  $L\frac{di}{dt}$  effects

increase too. Both these effects increase the voltage in the conductors undergoing commutation. However, the inter-pole flux increases too, producing a larger voltage in the conductors that opposes the voltage due to the neutral-plane shift. The net result is that their effects cancel over a broad range of loads. The inter-poles must induce a voltage in the conductors undergoing commutation that is opposite to the voltage

caused by neutral-plane shift and  $L\frac{di}{dt}$  effects. Note that inter-poles work for both

motor and generator operation. The current both in its rotor and in its inter-poles reverses direction. The inter-poles must be of the same polarity as the next upcoming main pole in a generator. The inter-poles must be of the same polarity as the previous main pole in a motor.



Fig. 2-17 DC machine with inter-poles

#### 2-6-3 Compensating winding:

For very heavy, severe duty cycle motors, the flux-weakening problem can be very serious. To completely cancel armature reaction and thus eliminate both neutral-plane shift and flux weakening, a different technique was developed. This third technique involves placing compensating windings in slots carved in the faces of the poles parallel to the rotor conductors, to cancel the distorting effect of armature reaction as shown in Fig. 2-18. These windings are connected in series with the rotor windings, so that whenever the load changes in the rotor, the current in the compensating windings changes, too. Notice that the magnetomotive force due to the rotor at every point under the pole faces. The resulting net magnetomotive force is just the magnetomotive force due to the poles, so the flux in the machine is unchanged regardless of the load on the machine. The major disadvantage of compensating windings is that they are expensive, since they must be machined into the faces of the poles.



Fig. 2-18 The flux and magnetomotive forces in a DC machine with compensating windings.

Any motor that uses them must also have inter-poles, since compensating windings do di

not cancel  $L\frac{di}{dt}$  effects. The inter-poles do not have to be as strong, though, since they

are canceling only  $L\frac{di}{dt}$  voltages in the windings, and not the voltages due to neutral-

plane shifting. Because of the expense of having both compensating windings and interpoles on such a machine, these windings are used only where the extremely severe nature of a motor 's duty demands them.

# 2-7 The Final Form of the Output Voltage Resulting from DC Machine:

The voltage in any single conductor under the pole faces is

$$\boldsymbol{e}_{ind} = \boldsymbol{v} \boldsymbol{B} \boldsymbol{l}$$

The output voltage of armature can be calculated as

$$E_{a} = \frac{Z}{a} v B l$$
 2.3

Where Z is the total number of conductors and a is the number of current path.

The total number of conductors can be calculated from the following relation

$$Z = 2C N_c$$
 2.4

Where C is the number of coils on the rotor and  $N_c$  is the number of turns per coil.

The number of current path for lap winding is

$$a = mP \tag{2.5}$$

Where m is plex of winding (1,2,3,5,....) and P is the number of poles for machine The number of current path for multiplex winding is

$$a = 2m \tag{2.6}$$

Due to the machine is rotating in circular path, the linear velocity can replaced as  $v = r_{\mathcal{O}_m}$  so the output voltage is

$$E_{a} = \frac{Z}{a} r \omega_{m} B l$$
 2.7

This voltage can be reexpressed in a more convenient form by noting that the flux of a pole is equal to the flux density under the pole times the pole's area:

$$\phi = B A_{p}$$
 2.8

The area of each pole can be expressed in term of total surface area due to rotor rotating as

$$A_{p} = \frac{A}{P}$$
 2.9

The surface area due to rotor rotating is

$$A = 2\Pi rl$$
 2.10  
By substituting from eqs. (2.8-2.10) into eq. 2.7 it is found that;

$$E_{a} = \left(\frac{ZP}{2\Pi a}\right) \phi_{\mathcal{O}_{m}}$$

$$E_{a} = K \phi_{\mathcal{O}_{m}}$$
Where  $K = \frac{ZP}{2\Pi a}$ 
2.11

From last equation, it is found that, the total induced voltage depends up on

- 1. The flux inside machine.
- 2. The rotating speed of the machine.
- 3. Constant depends upon the construction of the machine.

# 2-8 Induced Torque in Real DC Machine:

To drive the torque equation in real DC machine, it starts by the generating torque in one conductor of the DC machine. It is

$$\tau_{ind} = r i_{cond} l B \qquad 2.12$$
But  $i_{cond} = \frac{I_a}{a}$  so the torque equation becomes

$$\tau_{ind} = \frac{rI_a l B}{a}$$
 2.13

If the number of conductors are z so the torque equation becomes

$$\tau_{ind} = \frac{zrI_a l B}{a}$$
 2.14

But the flux under pole can be expressed as

$$\phi = B A_{P} = \frac{B(2\Pi rl)}{P}$$
 2.15

By substituting from eq. 2.15 into 2.14 it is found that

$$\tau_{ind} = (\frac{zP}{2\Pi a})\phi I_{a}$$
2.16
Let  $\frac{zP}{zP} = K$  so the real torque equation in the DC machine be

Let  $\frac{1}{2\Pi a} = K$  so the real torque equation in the DC machine becomes

$$\boldsymbol{\tau}_{ind} = \boldsymbol{K} \boldsymbol{\phi} \boldsymbol{I}_{a}$$
 2.17

From the last equation, it found the real torque equation in the DC machine depends upon

- 1. The flux inside the DC machine.
- 2. The DC machine current.
- 3. Constant depends upon the construction of the machine.

2-9 Solved Problem in the DC Machine:

Here some problems in the DC Machine will solve replace (2, 1):

Problem (2-1):

A duplex lap-wound armature is used in a six-pole DC machine with six brush sets, and carrying 20 Ampere each spanning two commutator segments. There are 72 coils on the armature, each containing 12 turns. The flux per pole in the machine is 0.039 Wb, and the machine spins at 400 RPM.

Calculate the following

- 1. The number of the current path in this machine.
- 2. The number of conductor in this machine.
- 3. The machine constant K.
- 4. Induced emf.
- 5. The torque induced in the DC machine.

Solution:

1. Calculating the number of the current path in this machine

The number of the current path = mP = 2x6 = 12 current path

2. Calculating the number of conductor in this machine.

 $Z = 2C N_c = 2x 72x 12 = 1728$ 

3. Calculating the machine constant

$$K = \frac{ZP}{2\Pi a} = \frac{1728x\,6}{2\Pi x\,12} = \frac{432}{\Pi}$$

4. Calculating the induced voltages in the machine

The induced voltages in the machine  $E_a = K \phi_{\mathcal{O}_m}$ 

$$E_a = \frac{432}{\Pi} x \, 0.039 x \, (\frac{2\Pi}{60} x \, 400) = 224.64 \text{ voltages}$$

5. The torque induced in the DC machine  $\tau_a = K \phi I_a$ 

$$\tau_a = \frac{432}{\Pi} x \, 0.039 x \, 20 = 134.14 \, \text{Nm}$$

#### Problem (2-2):

An 8-pole DC generator has 500 armature conductors, and a useful flux of 0.05 Wb per pole.

Calculate the following

- 1. The emf generated if it is lap-connected and runs at 1200 RPM.
- 2. The rotating speed for DC generator to obtain the same voltage in the above case if it is wave-wound connected.

Solution:

1- Calculating the emf inside the machine in case of lap winding

The number of the current path  $= mP = 1x \ 8 = 8$  current path

the machine constant =

$$K = \frac{ZP}{2\Pi a} = \frac{500x\,8}{2\Pi x\,8} = \frac{500}{2\Pi}$$

The induced voltages in the machine  $E_a = K \phi_{\mathcal{O}_m}$ 

$$E_a = \frac{500}{2\Pi} x \, 0.05 x \, (\frac{2\Pi}{60} x \, 1200) = 500 \, \text{voltages}$$

2- Calculating the rotating speed for DC generator to obtain the same voltage in the above case if it is wave-wound connected

The number of the current path = 2m = 2x 1 = 2

the machine constant =  $K = \frac{ZP}{2\Pi a} = \frac{500x \, 8}{2\Pi x \, 2} = \frac{1000}{\Pi}$ The rotor speed  $N_r = \frac{E_a}{K\phi(\frac{2\Pi}{60})} = \frac{500}{\frac{1000}{\Pi}x \, 0.05(\frac{2\Pi}{60})} = 300$  RPM

### DIRECT CURRENT GENERATORS

#### **<u>3-1 Introduction:</u>**

Direct current generators are used to convert the mechanical energy into electrical energy. Direct current (DC) generators are used initially in the lightening. After that they are used to feed the train networks, they are used to feed the magnetic field for alternating current machines and they are used in many industrial applications. There are many types of DC generators. These types are usually classified according to the way in which their fields are excited. Generators may be divided into

- 1- Separately-excited generators.
- 2- Self-excited generators.
- 3- Permanent magnet DC generator

Separately-excited generators mean that the field magnets are energized from an independent external source of DC current.

Self-excited generators mean that the field magnets are energized by the current produced by the generators themselves. Due to residual magnetism, there is always present some flux in the poles. When the armature is rotated, some emf and hence some induced current is produced which is partly or fully passed through the field coils thereby strengthening the residual pole flux. There are three types of self-excited generators named according to the manner in which their field coils (or windings) are connected to the armature. These various types of dc generators differ in their terminal (voltage-current) characteristics, and therefore in the applications to which they are suited.

DC generators are compared by their voltages, power ratings, efficiencies, and voltage regulations. Voltage regulation (VR) is defined by the equation

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} x \, 100$$
 3.1

Where  $V_{nl}$  is no-load voltages and  $V_{fl}$  is full-load terminal voltages of generator.

It is a rough measure of the shape of the generator 's voltage-current characteristic-a positive voltage regulation means a drooping characteristic, and a negative voltage regulation means a rising characteristic. All generators are driven by a source of mechanical power, which is usually called the prime mover of the generator. A prime mover for a dc generator may be a steam turbine, a diesel engine, or even an electric motor. Since the speed of the prime mover affects the output voltage of a generator, and since prime movers can vary widely in their speed characteristics, it is customary to compare the voltage regulation and output characteristics of different generators, assuming constant speed prime movers.

In this chapter the following will explain

- 1. The equivalent circuits for DC generators.
- 2. The various types and characteristics of DC generators.
- 3. Parallel operation of the DC generators.

#### <u>3-2 The equivalent circuit for DC generators:</u>

The DC generator consists of stator and rotor. The stator circuit is called field circuit and the rotor circuit is called armature circuit. Armature circuit represented by voltage source,  $E_a$  and a resistor  $R_a$ . The brush voltage drop represented by battery,  $V_{brush}$  opposing the direction of current flow in the DC generator. The field coils (field circuit) produce the magnetic flux are represented by inductor  $L_f$  and resistor  $R_f$ . The separate resistor,  $R_{adjust}$  represents an external variable resistor used to control the amount of current

in the field circuit. The equivalent circuit of DC generator can be represented as shown in Fig. 3-1.



Fig. 3-1 Equivalent circuit of DC generator

Due to the brush voltage drop is a very tiny fraction of the generated voltage in a machine so the voltage drop may be left out or approximately included in the value of  $R_a$ . The internal resistor in the field coils is sometimes lump together with the variable resistor, and the total is called  $R_f$ . So the equivalent circuit of DC generator can be modified as shown in Fig. 3-2.



field circuit

Armature circuit

Fig. 3-2 Modified equivalent circuit of DC generator

## **<u>3-3 Types of Generators:</u>**

Generators are usually classified according to the way in which their fields are excited. Generators may be divided into

- 1. Separately-excited generators.
- 2. Self-excited generators.
- 3. Permanent magnet DC generator

To study the DC generators, the following characteristics must be taken into considerations:

a- Open circuit characteristic (OCC).

- b- Internal or total Characteristic.
- c- External characteristic.

So these characteristics are studied for each types of the DC generators.

# 3-3-1 Separately-excited DC generators:

Separately-excited DC generators are those whose field magnets are energized from an independent external source of DC current. The modified field and armature circuits can be represented as shown in Fig. 3-3.

By applying the KVL on the of the field circuit and armature circuit it is found that,

$$I_f = \frac{V_f}{R_f}$$
 3.2

 $V_t = E_a - I_a R_a$ 

3.3



Fig. 3-3 Separately excited DC generator

To more understanding for this machine some characteristics must be studied. These characteristics can be explained as the follows,

<u>3-3-1a Open circuit characteristic for separately-excited DC generators:</u>

Open circuit characteristic is also known as magnetic characteristic or no-load saturation characteristic. This characteristic shows the relation between generated emf at no load ( $E_a$ ) and the field current ( $I_f$ ) at a given fixed speed. The open circuit characteristic curve is just the magnetization curve and it is practically similar for all type of generators. The data for open circuit characteristic curve is obtained by operating the generator at no load and keeping a constant speed with using the circuit in Fig. 3.4. In this circuit, the field current is gradually increased and the corresponding terminal voltage is recorded. The connection arrangement to obtain open circuit characteristic curve is as shown in the Fig. 3-5. For shunt or series excited generators, the field winding is disconnected from the machine and connected across an external supply. Fig. a is the operation of generator for the first time and Fig. b means the operation of the generator after first time. In Fig. a, the curve a means, the first operation of generator with increasing the field current where curve b means the operation of the generator with decreasing the field current. The two curves aren't congruent due effect of the residual flux. The Fig. b in the same Fig. represents the no-load test under effect of different speed. Where in Fig. b it is found that,



Fig. 3-4 The circuit used to study the open circuit characteristic curve for separately excited DC generator.



Fig. 3-5 The open circuit characteristic curve for separately excited DC generator at different speed.

The emf equation of dc generator which prove in the previous chapter, it knows that  $E_a = K \phi_{Om}$ . It is found that, the generated emf should be directly proportional to field flux i.e. it is directly proportional to the field current. However, even when the field current is zero, some amount of emf is generated. It is represented by OA in the Fig. 3-5. This initially induced emf is due to the fact that there exists some residual magnetism in the field poles. Due to the residual magnetism, a small initial emf is induced in the armature. This initially induced emf aids the existing residual flux, and hence, increasing the overall field flux. This consequently increases the induced emf. Thus, the open circuit characteristic follows a straight line. However, as the flux density increases, the poles get saturated and the  $\phi$  becomes practically constant. Thus, even we increase the  $I_f$  further,  $\phi$  remains constant and hence,  $E_a$  also remains constant. Hence, the open circuit characteristic curve looks like the B-H characteristic.

### <u>3-3-1b Load saturation curve:</u>

The curve showing relation between the terminal voltage V and field current  $I_f$  when the generator is loaded, is known as Load Saturation Curve. The curve

can be deduced from the no-load saturation curve provided the values of armature reaction and armature resistance are known. While considering this curve, account is taken of the demagnetizing effect of armature reaction and the voltage drop in armature which are practically absent under no-load conditions. The no-load saturation curve of Fig. 3-5.a has been repeated in Fig. 3-6 on a base of field amp-turns (and not current) and it is seen that at no-load, the field amp-turns required for rated no-load voltage are given by *Oa*. Under load conditions, the voltage will decrease due to demagnetizing effect of armature reaction. This decrease can be made up by suitably increasing the



Fig. 3-6 Load saturation curve for separately excited DC generator

field amp-turns. Let *ac* represent the equivalent demagnetizing amp-turns per pole. Then, it means that in order to generate the same emf on load as at no-load, the field amp-turns/pole must be increased by an amount ac = bd. The point *d* lies on the curve *LS* which shows relation between the voltage *E* generated under *load* conditions and the field amp-turns. The curve *LS* is practically parallel to curve *Ob*. The terminal voltage *V* will be less than this generated voltage *E* by an amount = *Ia Ra* where *Ra* is the resistance of the armature circuit. From point *d*, a vertical line de = IaRa is drawn. The point *e* lies on the full-load saturation curve for the generator. Similarly, other points are obtained in the same manner and the full-load saturation curve *Mp* is drawn. The right-angled triangle *bde* is known as drop reaction triangle. Load saturation curve for half-load can be obtained by joining the mid-points of such lines as *mn* and *bd* etc. In the case of self-excited generators, load saturation curves are obtained in a similar way.

#### 3-3-1c Internal characteristic for separately-excited DC generators:

An internal characteristic curve in Fig. 3-7 shows the relation between the onload generated emf ( $E_a$ ) and the armature current ( $I_a$ ). The on-load generated emf  $E_a$  is always less than E<sub>0</sub> due to the armature reaction.  $E_a$ can be determined by subtracting the drop due to demagnetizing effect of armature reaction from no-load voltage E<sub>0</sub>. Therefore, internal characteristic curve lies below the open circuit characteristic curve.

#### <u>3-3-1d External characteristic for separately-excited DC generators:</u>

An external characteristic curve shows the relation between terminal voltage  $(V_t)$  and the load current  $(I_L)$ . Terminal voltage  $V_t$  is less than the generated emf  $E_a$  due to voltage drop in the armature circuit. Therefore, external

characteristic curve lies below the internal characteristic curve. External characteristics are very important to determine the suitability of a generator for a given purpose. Therefore, this type of characteristic is sometimes also called as performance characteristic or load characteristic. This relation can be seen in Fig. 3-7. If there is no armature reaction and armature voltage drop, the voltage will remain constant for any load current. Thus, the straight line AB in above figure represents the no-load voltage versus. load current  $I_{L}$ . Due to the demagnetizing effect of armature reaction, the on-load generated emf is less than the no-load voltage. The curve AC represents the on-load generated emf  $E_{a}$  versus. load current  $I_{L}$  i.e. internal characteristic (as  $I_{a} = I_{L}$  for a separately excited dc generator). Also, the terminal voltage is lesser due to ohmic drop occurring in the armature and brushes. The curve AD represents the terminal voltage versus. load current i.e. external characteristic.



Characteristics of separately excited DC generator

Fig. 3-7 The internal and total characteristic curve for separately excited DC generator.

<u>3-3-2 Terminal voltage control of the separately excited DC generator:</u>

The methods of the terminal voltage control of the separately excited DC generator can be deduced from the induced voltage equation  $E_a = K \phi \omega_m$  inside armature circuit where it found that;

- By increasing the prime over speed, the generator speed increases and hence, the induced emf increases which means that, an increase in the terminal voltages and Vaus versa i.e. by decreasing the prime over speed, the induced emf is decrease.
- 2- By increasing the field current, the magnetic field in the field circuit increases and hence, the induced emf increases which means that, an increase in the terminal voltages and Vaus versa.

### Problem 3-1:

The magnetization curve for a separate excited dc generator is shown in Fig. 3-8. The generator is rated at 6 kW, 120 V, 50 A, and 1800 r/min and is shown in Fig. 3-9. Its field current is rated at 5 A. The following data are known about the machine:

$$R_a = 0.18\Omega$$
  $R_f = 120\Omega$   $R_{adjust} = 0$  to  $30\Omega$   $V_f = 120V$   $N_f = 1000Tum / pole$ 

1- If this generator is operating at no load, what is the range of voltage adjustments that can be achieved by changing  $R_{adjust}$ ? assuming no armature reaction.

2- If the field rheostat is allowed to vary from 0 to 30  $\Omega$  and the generator's speed is allowed to vary from 1500 to 2000 r/min, what are the maximum and minimum no-load voltages in the generator? assuming no armature reaction.

3- If the variable resistor in this generator's field circuit is adjusted to  $16\Omega$  and the generator's prime mover is driving it at 1600 r/min, what is this generator's no-load terminal voltage?

4- What would its voltage be if a \$0-A load were connected to its terminals? Assume that the generator has compensating windings.

5- What would its voltage be if a  $\pm$ 0-A load were connected to its terminals but the generator does not have compensating windings? Assume that its armature reaction at this load is 450 A  $\cdot$  turns.

6- How much field current would be needed to restore the terminal voltage to its no-load value? (Assume that the machine has compensating windings.) What is the required value for the resistor  $R_{adjust}$  to accomplish this?



Fig. 3-8 Magnetization curve for a separate excited dc generator



Fig. 3-9 Equivalent circuit for separately excited dc generator Solution:

1- The of the armature voltage is no-load can be calculated as, If  $R_{adjust}$  is equal to zero the terminal voltage is maximum value and can be find as,

 $I_f = \frac{V_f}{R_f + R_{adjust}} = \frac{120}{24 + 0} = 5$  Ampere and from magnetizing curve, the induced emf ( $E_a$ ) = 129 Volt so the terminal voltages are 129 Volt due no current passing through the armature circuit.

If  $R_{adjust}$  is equal to maximum value the terminal voltage is minimum value and can be find as,

 $I_f = \frac{V_f}{R_f + R_{adjust}} = \frac{120}{24 + 30} = 2.22$  Ampere and from the magnetizing curve,

the induced emf ( $E_a$ ) = 87.4 Volt so the terminal voltages are 87.4 Volt due no current passing through the armature circuit. the range of voltage adjustments that can be achieved by changing  $R_{adjust}$  are from 87.4 to 129 volts

> 2- The maximum and minimum no-load voltages in the generator when generator speed vary from 1500 to 2000 r/min, this calculated at  $R_{adjust}$  varied from 0 to 30  $\Omega$  so with help of this rule

$$\frac{E_{a2}}{E_{a1}} = \frac{N_{r2}}{N_{r1}} SO \quad E_{a2} = \frac{N_{r2}}{N_{r1}} x E_{a1}$$

At 1500 rpm/min, with =  $0\Omega$  the terminal voltages can be calculated as

$$E_{a2} = \frac{N_{r2}}{N_{r1}} x E_{a1} = \frac{1500}{1800} x 129 = 107.5$$
 volt

At 1500 rpm/min, with = 30 the terminal voltages can be calculated as

$$E_{a2} = \frac{1500}{1800} x \, 87.4 = 72.83$$
 volt

At 2000 rpm/min, with =  $0\Omega$  the terminal voltages can be calculated as

$$E_{a2} = \frac{2000}{1800} \times 129 = 143.33$$
 volt

At 2000 rpm/min, with = 30 the terminal voltages can be calculated as

$$E_{a^2} = \frac{2000}{1800} x \, 87.4 = 97.11$$
 volt

3- The of the armature voltage at no-load with motor speed is 1800 rpm can be calculated and  $R_{\alpha\alpha\beta\mu\sigma\sigma}$  is equal to 16 the induced voltage can be find as,

$$I_f = \frac{V_f}{R_f + R_{adjust}} = \frac{120}{24 + 16} = 3$$
 Ampere and from magnetizing curve, the induced emf ( $E_a$ ) = 107 Volt. When the motor speed becomes 1600 rpm the

induced voltage at no- load can be calculated as,

$$E_{a2} = \frac{N_{r2}}{N_{r1}} x E_{a1} = \frac{1600}{1800} x 107 = 95.11$$
 Volt. So, the terminal voltage at no-load  
= 95.11 volt

4- To calculate the terminal voltage at motor drawing 40 ampere as With the same conditions in part three follows the following,

$$V_{t} = E_{a} - I_{a}R_{a} = 95.11 - 40x \, 0.18 = 87.91$$
 volt

5- To calculate the terminal voltage at motor drawing 40 ampere as

With the same conditions in part three but the compensated winding doesn't conclude follows the following,

 $I_f = 3 - \frac{450}{1000} = 2.55$  ampere from magnetizing curve the induced voltage becomes 97.5 volt, this voltage is at 1800 rpm but the generator running at 1600 rpm so the induced voltage at this speed can be calculated as,

$$E_{a2} = \frac{N_{r2}}{N_{r1}} x E_{a1} = \frac{1600}{1800} x 97.5 = 86.667$$
 volt

The terminal voltage becomes

$$V_{t} = E_{a} - I_{a}R_{a} = 86.667 - 40x \, 0.18 = 79.46$$
 volt  
6- To make terminal voltage 95.11 volt, the induced voltage at 1600

rpm becomes

$$V_{t} = E_{a} + I_{a}R_{a} = 95.11 + 40x \ 0.18 = 102.31 \quad \text{volt}$$
$$E_{a2} = \frac{N_{r2}}{N_{r1}} x E_{a1} = \frac{1800}{1600} x \ 102.31 = 115.1 \quad \text{volt, from the magnetization curve}$$

the field current becomes 3.5 ampere

To calculate the new value of  $R_{adjust}$ 

$$R_{adjust} = \frac{V_f}{I_f} - R_f = \frac{120}{3.5} - 24 = 10.3\Omega$$

### 3-2-2 DC Shunt generator:

In a shunt generator, the field flux is derived by connecting the field circuit directly across the terminals of the generator. The modified field and armature circuits can be represented as shown in Fig. 3-10. By applying the KVL on the of the field circuit and armature circuit at point A it is found that,

$$I_f = \frac{V_t}{R_f}$$
 3.4

$$V_t = E_a - I_a R_a \qquad 3.5$$

By applying the KCL on at point A it is found that,



Fig. 3-10 Equivalent circuit DC shunt generator

To more understanding for this machine some characteristics must be studied. These characteristics can be explained as the follows,

#### <u>3-2-2a Building up the voltage of DC shunt generator:</u>

One of the simplest forms of 'self-excited' generator is the shunt-wound machine; the connection diagram is shown in Fig. 3-11 with remove the load. The manner in which a self-excited generator manages to excite its own field and build a DC voltage across its armature is described with reference to Fig. 3-11 in the following steps:

- Assume that the generator starts from rest, i.e., prime-mover speed is zero.
   Despite a residual magnetism, the generated emf E, is zero.
- 2. As the prime-mover rotates the generator armature and the speed approaches rated speed, the voltage due to residual magnetism and speed increases.
- 3. At rated speed, the voltage across the armature due to residual magnetism is small, E<sub>1</sub> as shown in the Fig. 3-12. But this voltage is also across the

field circuit whose resistance is  $R_f$  Thus, the current which flows in the field circuit  $I_1$ , is also small.

- 4. When  $I_1$  flows in the field circuit of the generator of Fig. 3-11, an increase in mmf results increasing the induced voltage to  $E_2$  as shown in Fig. 3-12.
- 5. Voltage  $E_2$  is now impressed across the field, causing a large current  $I_2$  to flow in the field circuit.  $I_2T_f$  is an increased mmf, which produces generated voltage  $E_3$ .
- 6. E<sub>3</sub> yields I<sub>3</sub> in the field circuit, producing E<sub>4</sub>. But E<sub>4</sub> causes I<sub>4</sub> to flow in the field producing E<sub>5</sub> ;and so on, up to E<sub>8</sub>, the maximum value,
- 7. The process continues until that point where the field resistance line crosses. the magnetization curve in Fig. 3-12. Here the process stops. The induced voltage produced, when impressed across the field circuit, produces a current flow that in turn produces an induced voltage of the same magnitude. Ea. as shown in the Fig. 3-12.



Fig. 3-11 The practical circuit to study the open circuit characteristic curve for shunt DC generator.

The critical resistance:

• In the above description, a particular value of field resistance R<sub>f</sub> was used for building up of self-excited shunt generator. If the field resistance were reduced by means of adjusting the field rheostat of Fig, 3-11 to a lower

value say  $R_{f1}$  shown in Fig. 3-12, the build-up process would take place along field resistance line  $R_{f1}$  and build-up a somewhat higher value than  $E_a$ , i.e. the point where  $R_{f1}$  intersects the magnetization curve,  $E_9$ . Since the curve is extremely saturated in the vicinity of  $E_9$ , reducing the field resistance (to its limiting field winding resistance) will not increase the voltage appreciably. Conversely, increasing the field rheostat resistance and the field circuit resistance (to a value having a higher slope than  $R_f$  in the figure) will cause a reduction of the maximum value to which buildup can possibly occur.



Fig. 3-12 shunt DC generator build up voltage

• The field resistance may be increased until the field circuit reaches a critical field resistance. Field circuit resistance above the critical field resistance will fail to produce build-up.

This critical field circuit resistance,  $R_c$ , is shown as tangent to the saturation curve passing through the origin, O, of the axes of the curve of Fig. 3-12.

Thus, a field circuit resistance higher than  $R_c$  will produce an armature voltage of  $E_1$  approximately (and no more).

<u>3-2-2b Reasons for failure of DC shunt generator to build-up voltage</u>: The reasons why a self-excited generator may fail to build-up voltage are given below:

1. **No residual magnetism**. The start of the build-up process requires some residual magnetism in the magnetic circuit of the generator. If there is little or no residual magnetism, because of inactivity or jarring in shipment, no voltage will be generated that can produce field current. To overcome this difficulty, a separate source of direct current is applied to the field for a short pedal, of time and then removed. The magnetic field should now be sufficient to allow the voltage to build-up. The application of a separate source of direct current to the field 'flashing the field'.

2. Field connection reversed. The voltage generated due to residual magnetism is applied to the field. Current should flow in the field coils in such a direction as to produce lines of flux in the same direction as the residual flux. If the field connections are reversed, the lines of flux produced by the current flow will oppose the residual flux so that the generated voltage will decrease rather than increase when the field circuit is dosed, In this instance it is necessary to reverse the field connections with respect to the armature.

2. **Field circuit resistance too. high**. A field circuit resistance greater than critical value will prevent an appreciable build-up. At no load, resistance greater than the critical may be caused by the following:

- **Open field circuit connection**. The effects of an open circuit are apparent. The field circuit resistance is much greater than the critical value; hence generator will not build- up.
- **Dirty commutator**. A dirty commutator does not permit good contact between the brushes and the commutator. This poor contact shows up as a high resistance to the flow of current in the field circuit and produces the same effect as a high field circuit resistance

#### <u>3-2-2c Characteristics of DC shunt generator:</u>

To determine the internal and external load characteristics of a DC shunt generator, the machine is allowed to build up its voltage before applying any external load. To build up voltage of a shunt generator, the generator is driven at the rated speed by a prime mover. Initial voltage is induced due to residual magnetism in the field poles. The generator builds up its voltage as explained by the open circuit characteristic curve. When the generator has built up the voltage, it is gradually loaded with resistive load and readings are taken at suitable intervals. Connection arrangement is as shown in the Fig. 3-13.



Fig. 3-13 The circuit used to study the internal and external characteristic curve for shunt DC generator.

Unlike, separately excited DC generator, here,  $I_a \neq I_L$ . For a shunt generator,  $I_a = I_f + I_L$ . Hence, the internal characteristic can be easily transmitted to  $E_a$  versus.  $I_L$  by subtracting the correct value of  $I_f$  from  $I_a$ . During a normal running condition, when load resistance is decreased, the load current increases. But, as we go on decreasing the load resistance, terminal voltage also falls. So, load resistance can be decreased up to a certain limit, after which the terminal voltage drastically decreases due to excessive armature reaction at very high armature current and increased I<sup>2</sup>R losses. Hence, beyond this limit any further decrease in load resistance results in decreasing load current. Consequently, the external characteristic curve turns back as shown by dotted line in the Fig. 3-14.



Characteristics of DC shunt generator

Fig. 3-14 DC shunt generator characteristics

### <u>3-2-3 Voltage control for a DC shunt generator:</u>

There are two ways to control the voltage of a shunt generator. These ways are

**1.** Change the shaft speed  $\omega_m$  of the generator.

2. Change the field resistor of the generator, thus changing the field current. Changing the field resistor is the principal method used to control terminal voltage in real shunt generators. If the field resistor  $R_f$  is decreased, then the

field current 
$$I_f = \frac{V_t}{R_f}$$
 increases. When  $I_f$  increases, the machine's flux  $\phi$ 

increases, causing the internal generated voltage  $E_a$  to increase. The increase

in  $E_a$  causes the terminal voltage of the generator to increase as well.

## Problem 3-2:

The following figures give the OCC of a DC shunt generator at 300 r.p.m.

Field amperes	0	2	3	4	5	6	7
Armature volt	7.5	92	132	162	183	190	212

- 1- Plot the O.C.C. for 375 r.p.m. and determine the voltage to which the machine will excite if field circuit resistance is 40  $\Omega$ .
- 2- What additional resistance would have to be inserted in the field circuit to reduce the voltage to 200 volts at 375 r.p.m.?
- 3- Without this additional resistance, determine the load current supplied by the generator, when its terminal voltage is 200 V. Ignore armature reaction and assume speed to be constant. Armature resistance is  $0.4 \Omega$ .
- 4- Find the critical resistance and

Solution:

1. With neglecting the voltage drop across the armature resistance it is found that,

$$\frac{E_{a2}}{E_{a1}} = \frac{N_{r2}}{N_{r1}} SO \quad E_{a2} = \frac{N_{r2}}{N_{r1}} x E_{a1}$$
$$E_{a2} = \frac{375}{300} x E_{a1} = 1.25 E_{a1}$$

 $E_{a2} = 1.25 E_{a1}$  this rule is used to generate the voltage at 375 rpm where  $E_{a1}$  is voltage from the table in the problem and  $E_{a2}$  is wanted voltage

Field amperes	0	2	3	4	5	6	7
Armature volt	9.4	115	165	202.5	228.8	248.8	265

2. From the open circuit characteristics and terminal voltage line, the noload voltage is 256 volts. This is shown in Fig. 3- 15. The value of add resistance to the field circuit to reduce the terminal voltage into 200 volt can be found that, from the open circuit characteristics and when



Fig. 3-15 Open loop DC shunt generator characteristics

terminal voltage line is 200 volt, the field current is 3.85 ampere so the field current becomes

$$R_{f} = \frac{V_{t}}{I_{f}} = \frac{200}{3.85} = 51.94\Omega$$
$$R_{add} = 51.94 - 40 = 11.94\Omega$$

1. To calculate the load current at terminal voltages 200 volt

The field current  $I_f = \frac{V_t}{R_f} = \frac{200}{40} = 5$  ampere from the open circuit

characteristics the induced voltage line is 228 volt,

$$I_{a} = \frac{E_{a} - V_{t}}{R_{a}} = \frac{228 - 200}{0.4} = 70$$
 ampere

 $I_{L} = I_{a} - I_{f} = 70 - 5 = 65$  ampere

2. The critical resistance can be find from critical line on the graph

$$R_{c}=\frac{225}{4}=45\Omega$$

### 3-2-3 DC Series generator:

In a series generator, the field flux is produced by connecting the field circuit in series with the armature of the generator. The modified field and armature circuits can be represented as shown in Fig. 3-16.



Fig. 3-16 Equivalent circuit DC series generator

By applying the KVL on the of the field circuit and armature circuit it is found that,

$$V_t = E_a - I_a (R_a + R_s)$$
3.7

To more understanding for this machine some characteristics must be studied. These characteristics can be explained as the follows,

## 3-3-3c Characteristic of series-generators:

These characteristics are made by circuit in Fig. 3-17. The curve AB in Fig. 3-18 identical to open circuit characteristic curve. This is because in DC series generators field winding is connected in series with armature and load.



Fig. 3-17 The circuit diagram for internal and external characteristics



Characteristics of DC series generator

Fig. 3-18 DC series generator characteristics

Hence, here load current is similar to field current (i.e.  $I_L = I_s$ ). The curve OC and OD represent internal and external characteristic respectively. In a DC series generator, terminal voltage increases with the load current. This is because, as the load current increases, field current also increases. However, beyond a certain limit, terminal voltage starts decreasing with increase in load. This is due to excessive demagnetizing effects of the armature reaction.

### Problem 3-3:

Series generator has 4 poles, 15 kw, 0.150hm field resistance, 0.24 armature resistance running by 1150 rpm at 125volt.

find the terminals voltage at 1200 rpm and feeding current 125ampere where the magnetic field increased by 30%

Solution:

The motor current = 
$$I_{L} = \frac{P_{0}}{V_{t}} = \frac{1500}{125} = 120$$
 ampere

The induced voltage at rated can be calculated as

$$E_{a} = V_{t} + I_{L}(R_{a} + R_{s}) = 125 + 120(0.24 + 0.15) = 171.8$$
 Volt

The induced voltage at 1200 rpm with magnetic field increased by 30%

$$= \frac{E_{a2}}{E_{a1}} = \frac{K\phi_2 N_{r2}}{K\phi_1 N_{r1}} \quad SO \quad E_{a2} = \frac{\phi_2 N_{r2}}{\phi_1 N_{r1}} \times E_{a1}$$
$$E_{a2} = \frac{1.3\phi_1 N_{r2}}{\phi_1 N_{r1}} \times E_{a1} = E_{a2} = \frac{1.3x1200}{1150} \times 171.8 = 233.05 \text{ Volt}$$
$$V_{t} = E_{a} - I_{L}(R_{a} + R_{s}) = 233.05 - 125(0.24 + 0.15) = 184.3 \text{ Volt}$$
3-2-4 Cumulatively compounded generator:

In series wound generators, the output voltage is directly proportional with load current. In shunt wound generators, output voltage is inversely proportional with load current. A combination of these two types of generators can overcome the disadvantages of both. This combination of windings is called compound wound DC generator. Compound wound generators have both series field winding and shunt field winding. One winding is placed in series with the armature and the other is placed in parallel with the armature. This type of DC generators may be of two types- differentially compounded generator and cumulatively compounded generator. In a compound wound generator, the shunt field is stronger than the series field. When the series field assists the shunt field, generator is said to be commutatively compound wound. On the other hand if series field opposes the shunt field, the generator is said to be differentially compound wound. The connections of these coils for these types under poles are shown in Fig. 3-19. The winding of these types can be connected as short shunt compound wound generator and long shunt compound wound generator. In a cumulatively compounded generator, both a shunt and a series field are present, and their effects are additive and this is shown in Fig. 3-20 and in Fig. 3-21. In a differentially compounded generator, both a shunt and a series field are present, but their effects are subtractive and this is shown in Fig. 3-22 and in Fig. 3-23. In both the cases the external characteristic of the generator will be nearly same. Differentially compounded generator wound generators are very rarely used while the cumulatively compounded generator is more spread.



Fig. 3-19 Cumulatively compound differentially compound under poles



Fig. 3-20 Long shunt cumulatively DC compounded generator



Fig. 3-21 Short shunt cumulatively DC compounded generator



Fig. 3-22 Long shunt differentially DC compounded generator



Fig. 3-23 Short shunt differentially DC compounded generator

By applying Kirchhoff current low on the long shunt cumulatively DC compounded generator, it is found that,

$$\boldsymbol{I}_{a} = \boldsymbol{I}_{f} + \boldsymbol{I}_{L} \qquad 3.8$$

By applying Kirchhoff current low on the long shunt cumulatively DC compounded generator, it is found that,

$$V_{t} = E_{a} - I_{a}(R_{a} + R_{s})$$
 3.9

The field current can be calculated as

$$I_f = \frac{V_f}{R_f}$$
 3.10

In cumulatively compounded DC generator which has long-shuntconnection, the dots that appear on the two field coils have the same meaning as the dots on a transformer: Current flowing into a dot produces a positive magnetomotive force. Notice that the armature current flows into the dotted end of the series field coil and that the shunt current  $I_f$  flows into the dotted end of the shunt field coil. Therefore, the total magnetomotive force on this machine is given by

$$\mathfrak{J}_{net} = \mathfrak{J}_f + \mathfrak{J}_{se} - \mathfrak{J}_{AR}$$

$$3.11$$

where  $\mathfrak{T}_{f}$  is the shunt field magnetomotive force.  $\mathfrak{T}_{se}$  is the series field magnetomotive force, and  $\mathfrak{T}_{AR}$  is the armature reaction magnetomotive force. The equivalent effective shunt field current for this machine is given by

$$N_{f}I_{f}^{*} = N_{f}I_{f} + N_{se}I_{a} - \mathfrak{T}_{AR}$$

$$I_{f}^{*} = I_{f} + \frac{N_{se}}{N_{f}}I_{a} - \frac{\mathfrak{T}_{AR}}{N_{f}}$$
3.12

By applying Kirchhoff current low on the short shunt cumulatively DC compounded generator, it is found that,

$$\boldsymbol{I}_{a} = \boldsymbol{I}_{f} + \boldsymbol{I}_{L} \qquad \qquad 3.13$$

By applying Kirchhoff current low on the short shunt cumulatively DC compounded generator, it is found that,

$$V_{t} = E_{a} - I_{a}R_{a} + I_{L}R_{s} \qquad 3.14$$

The field current can be calculated as

$$I_{f} = \frac{E_{a} - I_{a} R_{a}}{R_{f}}$$

$$3.15$$

In differentially compounded DC generator which has long-shuntconnection, the Kirchhoff current low and the Kirchhoff voltage low are the same for cumulatively compounded DC generator but the armature current is now flowing out of a dotted coil end, while the shunt field current is flowing into a dotted coil end. In this machine, the net magnetomotive force is given by

$$\mathfrak{T}_{net} = \mathfrak{T}_f - \mathfrak{T}_{se} - \mathfrak{T}_{AR}$$
 3.16

The equivalent effective shunt field current for this machine is given by

$$N_{f}I_{f}^{*} = N_{f}I_{f} + N_{se}I_{a} - \mathfrak{T}_{AR}$$

$$I_{f}^{*} = I_{f} - \frac{N_{se}}{N_{f}}I_{a} - \frac{\mathfrak{T}_{AR}}{N_{f}}$$
3.17

Fig. 3-24 shows the external characteristics of DC compound generators. If series winding amp-turns are adjusted so that, increase in load current causes increase in terminal voltage then the generator is called to be over compounded. The external characteristic for over compounded generator is shown by the curve AB in Fig. 3-24 If series winding amp-turns are adjusted so that, the terminal voltage remains constant even the load current is

increased, then the generator is called to be flat compounded. The external characteristic for a flat compounded generator is shown by the curve AC. If the series winding has lesser number of turns than that would be required to be flat compounded, then the generator is called to be under compounded. The external characteristics for an under compounded generator are shown by the curve AD.



Fig. 3-24 The external characteristics of DC compound generators <u>3-2-4-1Voltage control of cumulatively and differentially compounded DC generators:</u>

The techniques available for controlling the terminal voltage of a cumulatively and differentially compounded de generators are exactly the same as the techniques for controlling the voltage of a shunt dc generator i.e. the generator speed and field current.

Problem 3-4:

Long shunt cumulatively DC compounded generator feeds load has 60 ampere at 230 volt find the terminal voltage if armature resistance is 0.04 ohm, series resistance is 0.01 ohm and shunt resistance is 125 ohm

Solution:

$$I_{f} = \frac{V_{t}}{R_{f}} = \frac{230}{125} = 1.84 \text{ ampere}$$

$$I_{a} = I_{f} + I_{L} = 60 + 1.84 = 61.84 \text{ ampere}$$

$$E_{a} = V_{t} + I_{a}(R_{a} + R_{s}) = 230 + 61.84(0.01 + 0.04) = 233.1 \text{ volt}$$

## 3-2-5 Permanent magnet DC generator:

When the flux in the magnetic circuit is established by the help of permanent magnets then it is known as Permanent magnet DC generator Fig. 3-25. It consists of an armature and one or several permanent magnets situated around the armature. This type of dc generators generates very low power. So, they are rarely found in industrial applications. They are normally used in small applications like dynamos in motor cycles.



Fig. 3-25 Permanent magnet DC generator

## **3-3 Energy Losses and Efficiency of DC Generators:**

for a dc generator, input power is in the form of mechanical and the output power is in the form of electrical. In a practical machine, whole of the input power cannot be converted into output power as some power is lost in the process. This causes the efficiency of the machine to be reduced. Efficiency is the ratio of output power to the input power. Thus, in order to design rotating dc machines with higher efficiency, it is important to study the losses occurring in them. Various losses in a rotating DC generator can be characterized as follows:

### 3-3-1 Losses in a rotating DC machine:

- Copper losses
  - Armature Cu loss
  - Field Cu loss
  - Loss due to brush contact resistance
- Iron Losses
  - Hysteresis loss
  - Eddy current loss
- Mechanical losses
  - Friction loss
  - Windage loss

The above tree categorizes various types of losses that occur in a DC generator. Each of these is explained in details below.

### <u>3-3-1a Copper losses:</u>

These losses occur in armature and field copper windings. Copper losses consist of Armature copper loss, Field copper loss and loss due to brush contact resistance.

Armature copper loss =  $I_a^2 R_a$ 

This loss contributes about 30 to 40% to full load losses. The armature copper loss is variable and depends upon the amount of loading of the machine.

Field copper loss =  $I_f^2 R_f$ 

In the case of a shunt wounded field, field copper loss is practically constant. It contributes about 20 to 30% to full load losses.

Brush contact resistance also contributes to the copper losses. Generally, this loss is included into armature copper loss.

### 3-3-1b Iron losses (core losses)

As the armature core is made of iron and it rotates in a magnetic field, a small current gets induced in the core itself too. Due to this current, eddy current loss and hysteresis loss occur in the armature iron core. Iron losses are also called as Core losses or magnetic losses.

Hysteresis loss is due to the reversal of magnetization of the armature core. When the core passes under one pair of poles, it undergoes one complete cycle of magnetic reversal. The frequency of magnetic reversal if given by, f=P.N/120

where, P is number of poles and N is generator speed in rpm The loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. Hysteresis loss is given by,

Steinmetz formula:

 $W_h = \eta B_{max}^{1.6} fV$  (watts)
where,  $\eta$  = Steinmetz hysteresis constant and V = volume of the core in m<sup>3</sup> Eddy current loss:

When the armature core rotates in the magnetic field, an emf is also induced in the core (just like it induces in armature conductors), according to the Faraday's law of electromagnetic induction. Though this induced emf is small, it causes a large current to flow in the body due to the low resistance of the core. This current is known as eddy current. The power loss due to this current is known as eddy current loss.

### 3-3-1d Mechanical losses

Mechanical losses consist of the losses due to friction in bearings and commutator. Air friction loss of rotating armature also contributes to these. These losses are about 10 to 20% of full load losses. <u>Stray Losses:</u>

In addition to the losses stated above, there may be small losses present which are called as stray losses or miscellaneous losses. These losses are difficult to account. They are usually due to inaccuracies in the designing and modeling of the machine. Most of the times, stray losses are assumed to be 1% of the full-load.

# 3-3-2 Power flow diagram:

The most convenient method to understand these losses in a DC generator or a dc motor is using the power flow diagram. The diagram visualizes the amount of power that has been lost in various types of losses and the amount of power which has been actually converted into the output. Following are the typical power flow diagrams for a DC generator.

73



Fig. 3-26 power flow diagram for DC generator

The electrical efficiency can be calculated from the following relation

$$\eta_e = \frac{P_{out}}{P_{primover} - P_{stray}}$$

The mechanical efficiency can be calculated from the following relation

$$\eta_m = \frac{P_{primover} - P_{stray}}{P_{primover}}$$

The commercial efficiency can be calculated from the following relation

$$\eta_{c} = \frac{P_{out}}{P_{primover}}$$

# Problem 3-4:

A shunt generator delivers 195 A at terminal p.d. of 250 V. The armature resistance and shunt field resistance are 0.02  $\Omega$  and 50  $\Omega$  respectively. The iron and friction losses equal 950 W. Find

(a) E.M.F. generated (b) Cu losses (c) output of the prime motor

(d) commercial, mechanical and electrical efficiencies.

Solution:

To calculate E.M.F. generated

$$I_{f} = \frac{V_{f}}{R_{f}} = \frac{250}{50} = 5$$

$$I_{a} = I_{f} + I_{L} = 195 + 5 = 200$$

$$E_{a} = V_{f} + I_{a}R_{a} = 250 + 200x \ 0.02 = 254$$

To calculate Cu losses

Culosses = armature loss+ field loss

Cu losses =  $I_{f}^{2}R_{f} + I_{a}^{2}R_{a} = 5^{2}x \, 50 + 200^{2}x \, 0.02 = 2050$ 

To calculate the output of the prime motor

The output of the prime motor = output power +cooper loss +stray loss The output of the prime motor =

$$V_{t}I_{L} + P_{cu} + stray \ losses = 250x195 + 2050 + 950 = 51750$$

To calculate the mechanical, electrical, and commercial efficiencies respectively.

$$\eta_{m} = \frac{P_{primover} - P_{stray}}{P_{primover}} = \frac{51750 - 950}{51750} \times 100 = 98.2\%$$
$$\eta_{e} = \frac{P_{out}}{P_{primover} - P_{stray}} = \frac{48750}{51750 - 950} \times 100 = 95.9\%$$

$$\eta_{c} = \frac{P_{out}}{P_{primover}} = \frac{48750}{51750} \times 100 = 94.2\%$$

#### 3-4 Parallel Operation of D.C. Generators:

In a DC power plant, power is usually supplied from several generators of small ratings connected in parallel instead of from one large generator. This is due to the following reasons:

# (i) Continuity of service:

If a single large generator is used in the power plant, then in case of its breakdown, the whole plant will be shut down. However, if power is supplied from a number of small units operating in parallel, then in case of failure of one unit, the continuity of supply can be: maintained by other healthy units.

# (ii) Efficiency:

Generators run most efficiently when loaded to their rated capacity. Electric power costs less per kWh when the generator producing it is efficiently loaded. Therefore, when load demand on power plant decreases, one or more generators can be shut down and the remaining units can be efficiently loaded.

#### (iii) Maintenance and repair:

Generators generally require routine-maintenance and repair. Therefore, if generators are operated in parallel, the routine or emergency operations can be performed by isolating the affected generator while load is being supplied by other units. This leads to both safety and economy.

# (iv) Increasing plant capacity:

In the modern world of increasing population, the use of electricity is

continuously increasing. When added capacity is required, the new unit can be simply paralleled with the old units. In many situations, a single unit of desired large capacity may not be available. In that case a number of smaller units can be operated in parallel to meet the load requirement. Generally, a single large unit is more expensive.

# (v) Non-availability of single large unit:

In many situations, a single unit of desired large capacity may not be available. In that case a number of smaller units can be operated in parallel to meet the load requirement. Generally, a single large unit is more expensive.

#### 3-4-1 Connecting shunt generators in parallel:

The generators in a power plant are connected in parallel through bus-bars. The bus-bars are heavy thick copper bars and they act as +ve and - ve terminals. The positive terminals of the generators are connected to the +ve side of bus-bars and negative terminals to the negative side of bus-bars.



Fig. 3-27 two shunt generators connected by bus-bar Fig. 3-27 shows shunt generator 1 connected to the bus-bars and supplying load. When the load on the power plant increases beyond the capacity of this generator, the second shunt generator 2 is connected in parallel wish the first to meet the increased load demand. The procedure for paralleling generator 2 with generator 1 is as under:

(i) The prime mover of generator 2 is brought up to the rated speed. Now switch S4 in the field circuit of the generator 2 is closed.

(ii) Next circuit breaker CB-2 is closed and the excitation of generator2 is adjusted till it generates voltage equal to the bus-bars voltage. This isindicated by voltmeter V2.

(iii) Now the generator 2 is ready to be paralleled with generator 1. The main switch S3, is closed, thus putting generator 2 in parallel with generator 1. Note that generator 2 is not supplying any load because its generated e.m.f. is equal to bus-bars voltage. The generator is said to be "floating" (i.e., not supplying any load) on the bus-bars.

(iv) If generator 2 is to deliver any current, then its generated voltage E should be greater than the bus-bars voltage V. In that case, current

supplied by it is  $I_a = \frac{E_a - V_r}{R_a}$  where Ra is the resistance of the armature circuit. By increasing the field current (and hence induced e.m.f. E), the generator 2 can be made to supply proper amount of load.

(v) The load may be shifted from one shunt generator to another merely by adjusting the field excitation. Thus if generator 1 is to be shut down, the whole load can be shifted onto generator 2 provided it has the capacity to supply that load. In that case, reduce the current supplied by generator 1 to zero (This will be indicated by ammeter A1) open C.B.-1 and then open the main switch S1.

#### 3-4-2 Load Sharing:

The load sharing between shunt generators in parallel can be easily regulated because of their drooping characteristics. The load may be shifted from one generator to another merely by adjusting the field excitation. Let us discuss the load sharing of two generators which have unequal no-load voltages.

> Let  $E_{a1}$ ,  $E_{a2}$  = no-load voltages of the two generators  $R_{a1}$ ,  $R_{a2}$  = their armature resistances

> > $V_{t}$  = common terminal voltage (Bus-bars voltage)

then 
$$I_{a1} = \frac{E_{a1} - V_{t}}{R_{a1}}$$
 and  $I_{a2} = \frac{E_{a2} - V_{t}}{R_{a2}}$ 

Thus current output of the generators depends upon the values of  $E_{a1}$  and  $E_{a3}$ . These values may be changed by field rheostats. The common terminal voltage (or bus-bars voltage) will depend upon (i) the emfs of individual generators and (ii) the total load current supplied.

It is generally desired to keep the bus-bars voltage constant. This can be achieved by adjusting the field excitations of the generators operating in parallel.

#### Compound Generators in Parallel:

Under-compounded generators also operate satisfactorily in parallel but over compounded generators will not operate satisfactorily unless their series fields are paralleled. This is achieved by connecting two negative brushes together as shown in Fig. 3-28 (i). The conductor used to connect these brushes is generally called equalizer bar. Suppose that an attempt is made to

operate the two generators in Fig. 3-28 (ii) in parallel without an equalizer bar. If, for any reason, the current supplied by generator 1 increases slightly, the current in its series field will increase and raise the generated voltage. This will cause generator 1 to take more load. Since total load supplied to the system is constant, the current in generator 2 must decrease and as a result its series field is weakened. Since this effect is cumulative, the generator 1 will take the entire load and drive generator 2 as a motor. Under such conditions, the current in the two machines will be in the direction shown in Fig. 3-28 (ii). After machine 2 changes from a generator to a motor, the current in the shunt field will remain in the same direction, but the current in the armature and series field will reverse. Thus the magnetizing action, of the series field opposes that of the shunt field. As the current taken by the machine 2 increases, the demagnetizing action of series field becomes greater and the resultant field becomes weaker. The resultant field will finally become zero and at that time machine 2 will short circuit machine 1, opening the breaker of either or both machines.



Fig. 3-28 (i) the emfs of individual generators and (ii) the total load current supplied

When the equalizer bar is used, a stabilizing action exist? and neither machine tends to take all the load. To consider this, suppose that current delivered by generator 1 increases [Fig. 3-28 (i)]. The increased current will not only pass through the series field of generator 1 but also through the equalizer bar and series field of generator 2. Therefore, the voltage of both the machines increases and the generator 2 will take a part of the load.

# Problem 3-5:

Two DC. generators are connected in parallel to supply a load of 1500 A. One generator has an armature resistance of 0.5  $\Omega$  and an emf of 400 V while the other has an armature resistance of 0.04  $\Omega$  and an emf of 440 V. The resistances of shunt fields are 100  $\Omega$  and 80  $\Omega$  respectively. Calculate the sharing currents supplied by individual generators and terminal voltage of the combination.

# Solution:

This problem can be drawing as Fig. 3-29 to helping in solution



Fig. 3-29 drawing for problem 3-5

The current drawing from generator 2 to load is 1500-  $I_{L1}$ , The field current for generator 1 is  $I_{f1} = \frac{V}{100}$  and the field current for generator 2 is  $I_{f2} = \frac{V}{80}$  and the armature current for generator 1 is  $I_{a1} = \frac{V}{100} + I_{L1}$  and the armature current for generator 2 is  $I_{a2} = \frac{V}{80} + 1500 - I_{L1}$ 

The terminal voltage can be calculated for each DC generator can be calculated as,

For generator 1 the terminal voltage is  $400 - (\frac{V}{100} + I_{L1})x 0.5$  and for generator 1 the terminal voltage is  $440 - (\frac{V}{80} + 1500 - I_{L1})x 0.04$  due to the two terminals are equal so by equating them the load current for generator 1 is 33.8 ampere and the load current for generator 2 is 1466.2 ampere also the terminal voltage is 381.2 volt

The output for generator  $1 = 381.2 \times 33.8 = 12880 \text{ w}$ 

The output for generator  $2 = 381.2 \times 1466.2 = 558900 \text{ w}$ 

# **<u>3-5 Uses of D.C. Generators</u>**

**1. Shunt generators** with field regulators are used for ordinary lighting and power supply purposes. They are also used for charging batteries because their terminal voltages are almost constant or can be kept constant.

**2. Series generators** are not used for power supply because of their rising characteristics. However, their rising characteristic makes them suitable for being used as boosters (Ex. 28.15) in certain types of distribution systems particularly in railway service.

# 3. Compound generators

The cumulatively-compound generator is the most widely used DC generator because its external characteristic can be adjusted for compensating the voltage drop in the line resistance. Hence, such generators are used for motor driving which require DC supply at constant voltage, for lamp loads and for heavy power service such as electric railways. The differential-compound generator has an external characteristic similar to that of a shunt generator but with large demagnetization armature reaction. Hence, it is widely used in arc welding where larger voltage drop is desirable with increase in current.

#### **DIRECT CURRENT MOTORS**

Electrical motors are considered the driving forces of modern industries would not be an exaggeration if we say it's the backbone of those industries. These motors are used to convert the electric power into mechanical power. So, in this chapter, one types of these motors are studied. This type is direct current motor. Direct current motors are used in many industrial applications where It is frequently used in traction, cranes, textile, steel rolling, paper industries, and cement industries. This is because it has many advantages as controlling speed ease, high torque especially at starting.

### **<u>4-1 Working Principle of DC Motor</u>:**

A motor is an electrical machine which converts electrical energy into mechanical energy. The principle of working of a DC motor is that "whenever

a current carrying conductor is placed in a magnetic field, it experiences a mechanical force". The direction of this force is given by Fleming's left hand rule Which states that If we stretch the first finger, second finger and thumb of our left hand to be perpendicular to each other and direction of magnetic field is represented by the first finger, direction of the current is represented by second finger then the thumb represents the direction of the force experienced by the current carrying conductor. This Fleming's left hand rule is illustrated in Fig. 4-1. The magnitude of this force is given by the following equation

$$F = BIL\sin\theta \qquad 4.1$$

Where, *B* is magnetic flux density, *I* is current, *L* is length of the conductor within the magnetic field and  $\theta$  is the angle between the magnetic flux density and length of conductor.



Fig. 4-1 Fleming left hand rule

The following Figs will help in understanding the working principle of a DC motor. When armature windings are connected to a DC supply, current sets up in the winding. Magnetic field may be provided by field winding (electromagnetism) or by using permanent magnets. In this case, current

carrying armature conductors experience force due to the magnetic field, according to the principle stated above. Commutator is made segmented to achieve unidirectional torque. Otherwise, the direction of force would have reversed every time when the direction of movement of conductor is reversed the magnetic field.

If Fleming left-hand rule is applied on the Fig. 4-2a it is found that, the current in the left-hand side of the armature conductor to be I, and current at right hand side of the armature conductor to be - I, because they are flowing in the opposite direction with respect to each other. Then the force on the left-hand side armature conductor is

$$F = BIL \sin 90^\circ = BIL \qquad 4.2$$

Similarly force on the right-hand side conductor is

$$F = -BIL \sin 90^{\circ} = -BIL$$

4.3



a



b

Fig. 4-2 Principle operation of DC motor

The force on either side is equal in magnitude but opposite in direction. and since the two conductors are separated by some distance 2r = width of the armature turn = D, the two opposite forces produce a rotational force or a torque that results in the rotation of the armature conductor. Now let's examine the expression of torque when the armature turn crate an angle of  $\alpha$  with its initial position. The torque produced is given by

Torque = force tangential to the direction of the armature rotation X distance  $T = FD \cos \alpha = BILD \cos \alpha$  4.4

Where,  $\alpha$  is the angle between the plane of the armature turn and the plane of reference or the initial position of the armature which is here along the direction of magnetic field. The presence of the term  $\cos \alpha$  in the torque equation very well signifies that unlike force the torque at all position is not the same. It in fact varies with the variation of the angle  $\alpha$ . To explain the variation of torque and the principle behind rotation of the motor let us do a step wise analysis.

**Step 1:** Initially considering the armature is in its starting point or reference position where the angle  $\alpha = 0$ .

#### $T = BILD \cos 0^{\circ} = BILD \qquad 4.5$

Since,  $\alpha = 0$ , the term  $\cos \alpha = 1$ , or the maximum value, hence torque at this position is maximum given by T = BILD. This high starting torque helps in overcoming the initial inertia of rest of the armature and sets it into rotation.

**Step 2:** Once the armature is set in motion, the angle  $\alpha$  between the actual position of the armature and its reference initial position goes on increasing in the path of its rotation until it becomes 90° from its initial position. Consequently, the term cos $\alpha$  decreases and also the value of torque. The torque in this case is given by  $T = BILD \cos \alpha$  which is less than BILD when  $\alpha$  is greater than 0°.

**Step 3:** In the path of the rotation of the armature a point is reached where the actual position of the rotor is exactly perpendicular to its initial position, i.e.  $\alpha = 90^{\circ}$ , and as a result the term  $\cos \alpha = 0$ . The torque acting on the conductor at this position is given by,

$$T = BILD \cos 90^{\circ\circ} = 0 \qquad 4.6$$

i.e. virtually no rotating torque acts on the armature at this instance. But still the armature does not come to a standstill, this is because of the fact that the operation of DC motor has been engineered in such a way that the inertia of motion at this point is just enough to overcome this point of null torque. Once the rotor crosses over this position the angle between the actual position of the armature and the initial plane again decreases and torque starts acting on it again.

#### **4-2 Back EMF in DC Motor:**

According to fundamental laws of nature, no energy conversion is possible until there is something to oppose the conversion. In case of generators this opposition is provided by magnetic drag, but in case of DC motors there are back emfs. When the armature of the motor is rotating, the conductors are also cutting the magnetic flux lines and hence according to the Faraday's law of electromagnetic induction, an emf induces in the armature conductors. The direction of this induced emf is such that it opposes the armature current I<sub>a</sub>. The magnitude of back emf can be given by the emf equation of DC generator. Magnitude of back emf is directly proportional to speed of the motor. Consider the load on a dc motor is suddenly reduced. In this case, required torque will be small as compared to the current torque. Speed of the motor will start increasing due to the excess torque. Hence, being proportional to the speed, magnitude of the back emf will also increase. With increasing back emf armature current will start decreasing. Torque being proportional to the armature current, it will also decrease until it becomes sufficient for the load. Thus, speed of the motor will regulate. On the other hand, if a DC motor is suddenly loaded, the load will cause decrease in the speed. Due to decrease in speed, back emf will also decrease allowing more armature current. Increased armature current will increase the torque to satisfy the load requirement. Hence, presence of the back emf makes a DC motor 'selfregulating'.

# 4-3 Types Of DC Motors:

DC motors are usually classified of the basis of their excitation configuration, as follows -

- Separately excited (field winding is fed by external source)
- Self-excited -
  - Series wound (field winding is connected in series with the armature)

- Shunt wound (field winding is connected in parallel with the armature)
- Compound wound -
  - Long shunt
  - Short shunt
- Permanent magnet DC motor

Generally, three characteristic curves are considered important for DC motors which are, (i) Torque versus armature current, (ii) Speed versus armature current and (iii) Speed versus torque. These are explained below each type of DC motor. These characteristics are determined by keeping the following two relations in mind.

$$T \alpha \phi I_a$$
 and  $N \alpha \frac{E_a}{\phi}$ 

These above relations can be studied for a DC motor, magnitude of the back emf is given by the same emf equation of a DC generator i.e.  $E_a = K \phi_{\mathcal{O}_m}$ . The stigmatic diagram of DC motor as it is the stigmatic of DC generator but it is different in the direction of the armature current. In the following sections, the DC motors types are studied

# **4-3-1 Separately Excited DC Motor:**

separately excited DC motors is used in many applications such as train and automotive traction applications. In separately excited DC motors, the field winding is supplied from a separate power source. That means the field winding is electrically separated from the armature circuit. The modified field and armature circuits can be represented as shown in Fig. 4-3.

By applying the KVL on the of the field circuit and armature circuit it is found that,

$$I_f = \frac{V_f}{R_f}$$
 4.7

$$V_t = E_a + I_a R_a$$

$$4.8$$



Fig. 4-3 Separately excited DC motor

The induced voltage in the DC motor is

$$E_{a} = K \phi_{\mathcal{O}_{m}} \qquad 4.9$$

Also, the motor torque in the DC motor is

$$\tau = K \phi I \qquad 4.10$$

### 4-3-1-1 Characteristics of separately excited DC motor:

To study the performance characteristics of separately excited DC motor three characteristic curves are considered important for DC this motor which are, (i) Torque versus armature current, (ii) Speed versus armature current and (iii) Speed versus torque. These are explained below

# <u>4-3-1-1a Motor torque versus armature current:</u>

The relation between the motor torque and armature current can be deduced from eq. 4.10. it is found that, the motor torque is directly proportional to the armature current. This characteristic can be shown in Fig. 4-4.



Fig. 4-4 Motor torque versus the armature current

#### 4-3-1-1-b Motor speed versus armature current:

The relation between the motor speed and armature current can be deduced from eq. 4.8 and eq. 4.9. where from eq. 4.8 it is found that, the armature current can be calculated as,

$$I_a = \frac{V_t - E_a}{R_a}$$
 4.11

By substituting from eq. 4.9 into eq. 4.11 it is found that

$$I_a = \frac{V_t - K\phi\omega_m}{R_a}$$
 4.12

From above eq. it is found that,

$$\omega_m = \frac{V_t - I_a R_a}{K\phi}$$
 4.13

From above eq. it is found that, the motor speed is inversely proportional to the armature current. This relation can be shown in Fig. 4-6.



Fig. 4-6 Motor speed versus the armature current

#### 4-3-1-1-c Motor speed versus motor torque:

The relation between the motor speed and motor current can be deduced by substituting from eq. 4.12 into eq. 4.10 where it is found that,

$$\tau = K\phi(\frac{V_t - K\phi\omega_m}{R_a}) = \frac{K\phi V_t}{R_a} - \frac{(K\phi)^2\omega_m}{R_a}$$
 4.14

From the above eq. it is found that,

$$\omega_m = \frac{V_{t}}{K\phi} - \frac{R_{a}}{(K\phi)^2} \tau$$

$$4.15$$

From above eq. it is found that, the motor speed is inversely proportional to the motor torque. This relation can be shown in Fig. 4-7.



Fig. 4-7 Motor speed versus the motor torque

#### **4-3-1-2 Speed control of separately excited DC motor:**

By studying eq. 4.15 it is found that, the separately excited DC motors can be control in the speed by variation the input voltage (terminal voltage) or by variation the magnetic field. To vary the input voltage, this occurs by inserting resistance in the armature circuit or by using silicon controlled rectifier. In case of inserting resistances in the armature circuit it found that, by increasing the value of inserted resistance, the motor torque decreases as shown in Fig. 4-8-a where by using silicon controlled rectifier to control the armature voltage by controlling the firing angle it is found that the motor speed is directly proportional to armature voltage. This can be seen in Fig. 4-8-b. to control the motor speed by using the field circuit, this means variation the magnetic field. The variation of the magnetic field means weakening field or strengthening the field. Weakening of field causes increase in speed of the motor this occurs by inserted resistance or by reducing the voltage by using silicon controlled rectifier while strengthening the field causes decreases the speed and this occurs only by increasing the field voltage by step up

transformer. Fig. 4-9 shows effect of inserted resistance in the field circuit on the motor speed control where it is found that by increasing the inserted resistance the motor speed increase the same effect can be seen when the voltage applied on the field circuit decrease.



a- Effect of inserted resistance in the armature circuit on the motor speed control

b- Effect of varying armature voltage by silicon controlled rectifier on the motor speed control

Fig. 4-8 speed control of separately excited DC motor by controlling in the armature circuit.



Fig. 4-9 speed control of separately excited DC motor by controlling in the field circuit.

#### **4-3-2 Shunt wound DC Motor:**

Shunt wound DC motor is used in many applications such as lathe machines, centrifugal pumps, fans, blowers, conveyors, lifts, weaving machine, spinning machines. In this motor type the field winding is connected in parallel with the armature winding. Hence, the full voltage is applied across the field winding. Shunt winding is made with a large number of turns and the resistance is kept very high (about 100 Ohm). It takes only small current which is less than 5% of the rated armature current. The modified field and armature circuits can be represented as shown in Fig. 4-10.

By applying the KVL on the of the field circuit and armature circuit it is found that,



$$V_t = E_a + I_a R_a$$

4.17



Fig. 4-11 Shunt wound DC motor

By applying the KCL on the of the field circuit and armature circuit it is found that,

$$I_L = I_f + I_a$$
 4.18

The induced voltage and motor torque eqs. in the shunt, wound DC motor are the same as in eq. 4.9 and eq. 4.10. The characteristics of this motor is similar to that, the characteristics of separately excited DC motor which discussed as above.

# Problem 4-1:

A 250 V DC shunt motor has an armature resistance of 0.6 ohms and a field resistance of 170 ohms. The machine operates on full load at 950 rpm with an armature current of 40 A. If the speed is to be decreased to 700 rpm, calculate the required value of series armature resistance when the load torque is:

- (a) Proportional to speed.
- (b) Constant (as in a lift).

# Solution:

(a) The torque is the product of the interaction between the magnetic flux and the armature current. However, in this case the magnetic flux is constant. Thus, a change in torque would necessitate a proportional change in armature current. So, in the first case, where the torque is proportional to speed, then the armature current is also proportional to speed.

From the terminal voltage equation, it is found that;

$$V_{t} = E_{a} + I_{a} R_{a}$$
  

$$E_{a} = 250 - 40x \ 0.6 = 226$$
  

$$K\phi = \frac{E_{a}}{\omega_{m}} = \frac{226}{2\Pi x \ 950} x \ 60 = 2.27287$$

The rated load torque is

$$\tau = K \phi \mathbf{I}_{a}$$
N.m  
$$\tau = 2.27287X \ 40 = 90.915$$

To find the torque at motor speed is equal to 700 rpm

$$\tau \alpha_{\mathcal{W}_m}$$

$$\frac{\tau_2}{\tau_1} = \frac{K \phi_{\mathcal{W}_2}}{K \phi_{\mathcal{W}_1}} = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1}$$

$$\tau_2 = \frac{N_2}{N_1} \tau_1 = \frac{700}{950} \times 90.915 = 67$$
N.m

The armature current at the motor torque is

$$I_{a} = \frac{\tau_{2}}{K\phi} = \frac{67}{2.27287} = 29.478$$
 ampere

The induced voltage is

$$E_{a2} = K \phi_{\mathcal{O}_{m2}} = 2.27287 x \frac{2\Pi}{60} x 700 = 166.525$$
 Volt

The value of inserted resistance in the armature circuit is calculated as

$$R_{a} + R_{inserted} = \frac{V_{t} - E_{a2}}{I_{a2}}$$

$$R_{inserted} = \frac{V_{t} - E_{a2}}{I_{a2}} - R_{a}$$
Ohm
$$R_{inserted} = \frac{250 - 166.525}{29.478} - 0.6 = 2.2317$$

(b) When the torque is constant, then the armature current would not drop as the speed drops. Thus, the equation would be:

The value of inserted resistance in the armature circuit is calculated as

$$R_{a} + R_{inserted} = \frac{V_{t} - E_{a2}}{I_{a1}}$$

$$R_{inserted} = \frac{V_{t} - E_{a2}}{I_{a2}} - R_{a}$$
Ohm
$$R_{inserted} = \frac{250 - 166.525}{40} - 0.6 = 1.486875$$

#### 4-3-3 Series wound DC Motor:

In the series wound DC motor, the field winding is connected in series with the armature winding. Therefore, the field winding carries whole load current (armature current). That is why series winding is designed with few turns of thick wire and the resistance is kept very low (about 0.5 Ohm). field flux is produced by connecting the field circuit in series with the armature of the motor. The modified field and armature circuits can be represented as shown in Fig. 4-12.



Fig. 4-12 Equivalent circuit DC series motor

By applying the KVL on the of the field circuit and armature circuit it is found that,

$$V_t = E_a + I_a (R_a + R_s)$$
 4.19

To more understanding for this machine some characteristics must be studied. These are, (i) Torque versus armature current, (ii) Speed versus armature current and (iii) Speed versus torque. These are explained below

<u>4-3-3-1 Motor torque versus armature current:</u>

This characteristic is also known as electrical characteristic. The terminal characteristic of a series de motor is very different from that of the shunt motor previously studied. The basic behavior of a series dc motor is due to the fact that the flux is directly proportional to the armature current. As the load on the motor increases, its flux increases too. As seen earlier, an increase in flux in the motor causes a decrease in its speed. The result is that a series motor has a sharply drooping torque-speed characteristic. The induced torque in this machine is given by eq. 4.10. Also, the flux in this machine is directly proportional to its armature current (at least until the metal saturates). Therefore, the flux in the machine can be given by

$$\phi = c I_a \qquad 4.20$$

By substituting from eq. 4.20 into 4.10 it is found that

$$\tau = Kc I_a^{2} \qquad 4.21$$

From the last eq. it is found that, the torque in the motor is proportional to the square of its armature current as shown in Fig. 4-13.



Fig. 4-13 Motor torque versus the armature current

As a result of this relationship, it is easy to see that a series motor gives more torque per ampere than any other de motor. It is therefore used in applications requiring very high torques. Examples of such applications are the starter motors in cars, elevator motors, and tractor motors in locomotives,

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<u>4-3-3-2 Motor speed versus the armature current:</u>
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To understanding this characteristic, the relation between motor speed and armature current can be deduced as the follows,

By substituting from eq. 4.20 into 4.9 it is found that,

$$E_{a} = Kc I_{a} \omega_{m} \qquad 4.22$$

By substituting from eq. 4.22 into 4.8 it is found that,

$$V_{t} = (R_{a} + R_{s})I_{a} + Kc I_{a}\omega_{m}$$

$$V_{t} = (R_{a} + R_{s} + Kc \omega_{m})I_{a}$$

$$I_{a} = \frac{V_{t}}{R_{a} + R_{s} + Kc \omega_{m}}$$
4.23

This means that, the motor speed is inversely proportional to the armature current as shown in Fig. 4-14 and this can be explained as the follows,



Fig. 4-14 Motor speed versus the armature current

For small load current, change in back emf is small and it may be neglected. Hence, for small currents speed is inversely proportional to flux. As we know, flux is directly proportional to armature current, speed is inversely proportional to armature current. Therefore, when armature current is very small the speed becomes dangerously high. That is why a series motor should never be started without some mechanical load. But, at heavy loads, armature current is large. And hence, speed is low which results in decreased back emf. Due to decreased more armature current is allowed.

#### <u>4-3-3-3 Motor speed versus the motor torque:</u>

This characteristic is also called as mechanical characteristic. To understanding this characteristic, the relation between motor speed and motor torque can be deduced as the follows,

By substituting from eq. 4.23 into 4.21 it is found that,

$$\tau = Kc \left(\frac{V_{t}}{R_{a} + R_{s} + Kc \omega_{m}}\right)^{2} = Kc \frac{V_{t}^{2}}{(R_{a} + R_{s} + Kc \omega_{m})^{2}}$$

$$(R_{a} + R_{s} + Kc \omega_{m})^{2} = Kc \frac{V_{t}^{2}}{\tau}$$

$$4.24$$

$$\omega_{m} = \frac{V_{t}}{\sqrt{Kc}\sqrt{\tau}} - \frac{R_{a} + R_{s}}{K}$$

From the above eq. 4.23 it is found that, the speed of the motor varies as the reciprocal of the square root of the torque as shown in Fig. 4-15. One disadvantage of series motors can be seen immediately from this equation. When the torque on this motor goes to zero, its speed goes to infinity. In practice, the torque can never go entirely to zero because of the mechanical, core, and stray losses that must be overcome. However, if no other load is

connected to the motor, it can turn fast enough to seriously damage itself. Never completely unload a series motor, and never connect one to a load by a belt or other mechanism that could break. If that were to happen and the motor were to become unloaded while running, the results could be serious.



Fig. 4-15 Motor speed versus the armature current

# **4-3-3-4 Speed control of series motor:**

The speed control of DC series motors can be obtained by two methods (i) flux control method (ii) armature control method. Armature control method is mostly used.

# 1. Flux control method:

The flux control method can be performed by field divertor, armature divertor, tapped field control and paralleling field coils

Field divertor:

A veritable resistance is connected parallel to the series field as shown in Fig. 4-14 a. This variable resistor is called as divertor, as the desired amount of

current can be diverted through this resistor and hence current through field coil can be decreased. Hence, flux can be decreased to the desired amount and speed can be increased.

#### Armature divertor:

Divertor is connected across the armature as in Fig. 4-14 b. For a given constant load torque, if armature current is reduced then flux must I increase. As, developed torque is directly proportional magnetic flux and armature current. This will result in an increase in current taken from the supply and hence flux will increase and subsequently speed of the motor will decrease. Tapped field control:

As shown in Fig. 4-14 c field coil is tapped dividing number of turns. Thus we can select different value of field flux by selecting different number of turns.

### Paralleling field coils:

In this method, several speeds can be obtained by regrouping coils as shown in Fig. 4-14 d.



Fig. 4-14 The speed control method of DC series motor by field control

# 2. Armature-resistance Control:

In this method, Speed can be controlled by armature control may be had by any one the following 2 methods: -

- 1. Armature resistance control
- 2. Armature terminal voltage control

# Armature resistance:

In this method, a variable resistance is directly connected in series with the supply to the complete motor as shown in Fig. 4-15. a variable resistance is directly connected in series with the supply. This reduces the voltage available across the armature and hence the speed falls. By changing the value of variable resistance, any speed below the normal speed can be obtained. This is the most common method employed to control the speed of DC series motors.



Fig. 4-15The speed control method of DC series motor by armature resistance control

although this method has poor speed regulation, this has no significance for series motors because they are used in varying speed applications. The loss of power in the series resistance for many applications of series motors is not too serious since in these applications, the control is utilized for a large portion of the time for reducing the speed under light-load conditions and is only used intermittently when the motor is carrying full-load.

#### Armature terminal voltage control:

The speed control of Dc series motor can be accomplished by supplying the power to the motor from a separate variable voltage supply. This method gives a large speed range with any desired no. of speed points. It is essentially a constant-torque system, because the output delivered by the motor decreases with a decrease in applied voltage and a corresponding decrease in speed.

#### 3. Series-parallel control:

This system is widely used in electric traction, where two or more mechanically coupled series motors are employed. For low speeds, the motors are connected in series, and for higher speeds the motors are connected in parallel. When in series, the motors have the same current passing through them, although voltage across each motor is divided. When in parallel, the voltage across each motor is same although the current gets divided. This shown in Fig. 4-16. The motor is started up in series with each other Additional resistance ( $\mathbf{R}$ ) is gradually cut-out by the controller when the motor attain speed then finally the control resistance is removed, then each motor has 1/2voltage of the line across it, this is the first running position. Here for any given value of armature current, each motor will run at half of its normal speed.



Fig. 4-16 Series-parallel control of DC series motor

Voltage across each motor = V/2

Current through each motor = I

Since there is no external resistance in the circuit, there is no waste of energy and so motors operate at efficiency nearly equal to that obtainable with full line voltage across the terminals of each motor. When it is desired to increase the speed of the combination, motors are connected in parallel and in series with a variable R. This resistance is gradually cut out as the motor attains speed and finally when this resistance is totally removed from the circuit as in fig. shown below, the second running position is obtained. In this position each motor is connected across the full line voltage.

Voltage across each motor = V

Current across each motor = I/2

# Problem 4-2:

A 220 V DC series motor has armature and field resistances of 0.15  $\Omega$  and 0.10  $\Omega$  respectively. It takes a current of 30 A from the supply while running at 1000 rpm. If an external resistance of 1  $\Omega$  is inserted in series with the motor, calculate the new steady state armature current and the speed. Assume the load torque remains constant.

### Solution:

To calculate the new steady state armature current, from the following eq. it is found the armature current doesn't change due to the load torque remain constant

$$\tau = Kc I_{a}^{2}$$

$$\tau \alpha I_{a}^{2}$$

$$\frac{\tau_{2}}{\tau_{1}} = \frac{I_{a2}^{2}}{I_{a1}^{2}} \quad due \quad to \tau_{1} = \tau_{2} \quad so I_{a2} = I_{a1} = 40 \text{ ampere}$$

To calculate the new steady state motor speed flows the following

$$E_{a} = Kc I_{a} \omega_{m}$$

$$\frac{E_{a2}}{E_{a1}} = \frac{Kc I_{a2} \omega_{m2}}{Kc I_{a1} \omega_{m1}} = \frac{\omega_{m2}}{\omega_{m1}}$$

$$\omega_{m2} = \frac{E_{a2}}{E_{a1}} \omega_{m1}$$
$$E_{a1} = V_{t} - I_{a1}(R_{a} + R_{s})$$

$$E_{a1} = 220 - 30(0.15 + 0.1) = 212.5$$

$$E_{a1} = 220 - 30(0.15 + 0.1 + 1) = 182.5$$

$$\omega_{m2} = \frac{E_{a2}}{E_{a1}} \omega_{m1}$$

$$N_{2} = \frac{E_{a2}}{E_{a1}} N_{1}$$

$$N_{2} = \frac{182.5}{212.5} \times 1000 = 858.82$$

### **<u>4-3-4 Compound-wound DC Motor:</u>**

DC compound motors have both series as well as shunt winding. There are two common types of compound motor connection, the long-shunt connection and short-shunt connection. In a compound motor, if series and shunt windings are connected such that series flux is in direction as that of the shunt flux then the motor is said to be cumulatively compounded Figs 4-17 and 4-18. And if the series flux is opposite to the direction of the shunt flux, then the motor is said to be differentially compounded Figs 4-19 and 4-20.



Fig. 4-17 Long shunt cumulatively DC compounded motor



Fig. 4-18 Short shunt cumulatively DC compounded motor



Fig. 4-19 Long shunt differentially DC compounded motor



Fig. 4-20 Short shunt differentially DC compounded motor

By applying Kirchhoff current low on the long shunt cumulatively DC compounded motor, it is found that,

$$\boldsymbol{I}_{L} = \boldsymbol{I}_{f} + \boldsymbol{I}_{a} \qquad 4.25$$

By applying Kirchhoff current low on the long shunt cumulatively DC compounded motor, it is found that,

$$V_{t} = E_{a} + I_{a} (R_{a} + R_{s})$$
4.26

The field current can be calculated as

$$I_{f} = \frac{V_{f}}{R_{f}}$$

$$4.27$$

In cumulatively compounded DC motor which has long-shunt connection, the dots that appear on the two field coils have the same meaning as the dots on a transformer: Current flowing into a dot produces a positive magnetomotive force. Notice that the armature current flows into the dotted end of the series field coil and that the shunt current  $I_f$  flows into the dotted end of the shunt field coil. Therefore, the total magnetomotive force on this machine is given by

$$\mathfrak{J}_{net} = \mathfrak{J}_f + \mathfrak{J}_{se} - \mathfrak{J}_{AR}$$

$$4.28$$

where  $\mathfrak{T}_{f}$  is the shunt field magnetomotive force.  $\mathfrak{T}_{se}$  is the series field magnetomotive force, and  $\mathfrak{T}_{AR}$  is the armature reaction magnetomotive force. The equivalent effective shunt field current for this machine is given by

$$N_{f}I_{f} = N_{f}I_{f} + N_{se}I_{a} - \mathfrak{T}_{AR}$$

$$I_{f}^{*} = I_{f} + \frac{N_{se}}{N_{f}}I_{a} - \frac{\mathfrak{T}_{AR}}{N_{f}}$$

$$4.29$$

By applying Kirchhoff current low on the short shunt cumulatively DC compounded motor, it is found that,

$$\boldsymbol{I}_{L} = \boldsymbol{I}_{f} + \boldsymbol{I}_{a} \qquad 4.30$$

By applying Kirchhoff current low on the short shunt cumulatively DC compounded motor, it is found that,

$$V_{t} = E_{a} + I_{a}R_{a} + I_{L}R_{s}$$
4.31

The field current can be calculated as

$$I_{f} = \frac{V_{f} - I_{s}R_{s}}{R_{f}}$$

$$4.32$$

In differentially compounded DC motor which has long-shunt- connection, the Kirchhoff current low and the Kirchhoff voltage low are the same for cumulatively compounded DC motor but the armature current is now flowing out of a dotted coil end, while the shunt field current is flowing into a dotted coil end. In this machine, the net magnetomotive force is given by

$$\mathfrak{T}_{net} = \mathfrak{T}_f - \mathfrak{T}_{se} - \mathfrak{T}_{AR}$$

$$4.33$$

The equivalent effective shunt field current for this machine is given by

$$N_{f}I_{f}^{*} = N_{f}I_{f} + N_{se}I_{a} - \mathfrak{T}_{AR}$$

$$I_{f}^{*} = I_{f} - \frac{N_{se}}{N_{f}}I_{a} - \frac{\mathfrak{T}_{AR}}{N_{f}}$$

$$4.34$$

### 4-3-4-1 Characteristics of DC compound motor:

Characteristics of both these compound motors Fig.4-21 are explained below.

# (a) Cumulative compound motor

Cumulative compound motors are used where series characteristics are required but the load is likely to be removed completely. Series winding takes care of the heavy load, whereas the shunt winding prevents the motor from running at dangerously high speed when the load is suddenly removed. These motors have generally employed a flywheel, where sudden and temporary loads are applied like in rolling mills.

(b) Differential compound motor

Since in differential field motors, series flux opposes shunt flux, the total flux decreases with increase in load. Due to this, the speed remains almost constant or even it may increase slightly with increase in load ( $N \propto E_a/\phi$ ). Differential compound motors are not commonly used, but they find limited applications in experimental and research work.



Fig. 4-21 Characteristics of compound DC motor

## 4-3-5 Permanent magnet DC motors:

Basic configuration of a permanent magnet DC motor is very similar to that of a normal DC motor. The working principle of any DC motor is same, i.e. when a current carrying conductor is placed in a magnetic field, it experiences a force. A permanent magnet DC motor also works on the same principle. In a PMDC motor, permanent magnets (located in stator) provide magnetic field, instead of stator winding. The stator is usually made from steel in cylindrical form. Permanent magnets are usually made from rare earth materials or neodymium. The rotor is slotted armature which carries armature winding. Rotor is made from layers of laminated silicon steel to reduce eddy current losses. Ends of armature winding are connected to commutator segments on which the brushes rest. Commutator is made from copper and brushes are usually made from carbon or graphite. DC supply is applied across these brushes. The commutator is in segmented form to achieve unidirectional torque. The reversal of direction can be easily achieved by reversing polarity of the applied voltage.

#### <u>4-3-5-1 Characteristics of PMDC motors:</u>

are similar to the characteristics of dc shunt motor in terms of torque, speed and armature current. However, speed-torque characteristics are more linear and predictable in PMDC motors. The diagrammatic representation of a permanent magnet DC motor is given below. The torque equation of DC motor suggests  $\tau = K \phi I_a$ . Here  $\phi$  is always constant, as permanent magnets of required flux density are chosen at the time of construction and can't be changed there after. For a permanent magnet DC motor  $\tau = Kx I_a$  Where,  $Kx = K \phi$  which is another constant. In this case the torque of DC Motor can only be changed by controlling armature supply.

## 4-3-5-2 Applications Of permanent magnet DC motors:

Permanent magnet dc motors are extensively used where smaller power ratings are required, e.g. in toys, small robots, computer disc drives etc.

#### 4-3-5-3 Advantages:

- 1. For smaller ratings, use of permanent magnets reduces manufacturing cost.
- 2. No need of field excitation winding, hence construction is simpler.

- 3. No need of electrical supply for field excitation, hence PMDC motor is relatively more efficient.
- 4. Relatively smaller in size
- 5. Cheap in cost

# 4-3-6-4 Disadvantages:

- 1. Since the stator in PMDC motor consists of permanent magnets, it is not possible to add extra ampere-turns to reduce armature reaction. Thus armature reaction is more in PMDC motors.
- 2. Stator side field control, for controlling speed of the motor, is not possible in permanent magnet dc motors.

# 4-4 Energy Losses and Efficiency of DC Generators:

for a dc motor, input power is in the form of electrical and output power is in the form of mechanical. In a practical machine, whole of the input power can not be converted into output power as some power is lost in the process. This causes the efficiency of the machine to be reduced. Efficiency is the ratio of output power to the input power. Thus, in order to design rotating dc machines with higher efficiency, it is important to study the losses occurring in them. Various losses in a rotating DC motor can be characterized as follows:

# 4-4-1 Losses in a rotating DC motor:

- Copper losses
  - Armature Cu loss
  - Field Cu loss
  - Loss due to brush contact resistance
- Iron Losses
  - Hysteresis loss
  - Eddy current loss

- Mechanical losses
  - Friction loss
  - Windage loss

The above tree categorizes various types of losses that occur in a DC generator. Each of these is explained in details below.

## 4-4-1a Copper losses:

These losses occur in armature and field copper windings. Copper losses consist of Armature copper loss, Field copper loss and loss due to brush contact resistance.

Armature copper loss = 
$$I_a^2 R_a$$
 4.35

This loss contributes about 30 to 40% to full load losses. The armature copper loss is variable and depends upon the amount of loading of the machine.

Field copper loss = 
$$I_f^2 R_f$$
 4.36

In the case of a shunt wounded field, field copper loss is practically constant. It contributes about 20 to 30% to full load losses.

Brush contact resistance also contributes to the copper losses. Generally, this loss is included into armature copper loss.

## 4-4-1b Iron losses (core losses)

As the armature core is made of iron and it rotates in a magnetic field, a small current gets induced in the core itself too. Due to this current, eddy current loss and hysteresis loss occur in the armature iron core. Iron losses are also called as Core losses or magnetic losses. Hysteresis loss is due to the reversal of magnetization of the armature core. When the core passes under one pair of poles, it undergoes one complete cycle of magnetic reversal. The frequency of magnetic reversal if given by, f=P.N/120 4.37

where, P is number of poles and N is generator speed in rpm The loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. Hysteresis loss is given by,

Steinmetz formula:

 $W_{h} = \eta B_{max}^{1.6} fV \text{ (watts)}$  4.38

where,  $\eta$  = Steinmetz hysteresis constant and V = volume of the core in m<sup>3</sup> Eddy current loss:

When the armature core rotates in the magnetic field, an emf is also induced in the core (just like it induces in armature conductors), according to the Faraday's law of electromagnetic induction. Though this induced emf is small, it causes a large current to flow in the body due to the low resistance of the core. This current is known as eddy current. The power loss due to this current is known as eddy current loss.

### 4-4-1d Mechanical losses

Mechanical losses consist of the losses due to friction in bearings and commutator. Air friction loss of rotating armature also contributes to these. These losses are about 10 to 20% of full load losses. <u>Stray Losses:</u>

In addition to the losses stated above, there may be small losses present which are called as stray losses or miscellaneous losses. These losses are difficult to account. They are usually due to inaccuracies in the designing and modeling of the machine. Most of the times, stray losses are assumed to be 1% of the full-load.

4-4-2 Power flow diagram:

The most convenient method to understand these losses in a DC motor is using the power flow diagram in Fig 4-22. The diagram visualizes the amount of power that has been lost in various types of losses and the amount of power which has been actually converted into the output. Following are the typical power flow diagrams for a DC motor.



Power flow diagram of a DC motor

Fig. 4-22 Power flow diagram of DC moto

The electrical efficiency can be calculated from the following relation

$$\eta_{e} = \frac{P_{driving}}{P_{input}}$$

$$4.39$$

The mechanical efficiency can be calculated from the following relation

$$\eta_{m} = \frac{P_{out}}{P_{driving}}$$
 4.40

The commercial efficiency can be calculated from the following relation

$$\eta_{c} = \frac{P_{out}}{P_{pin}}$$
 4.41

Where  $P_{input} = V_{I}I_{L}$  and  $P_{driving} = E_{a}I_{a}$ and  $P_{out} = P_{driving}$  -iron and fricition losses

### Problem 4-3:

A DC shunt machine while running as generator develops a voltage of 250 V at 1000 r.p.m. on no-load. It has armature resistance of 0.5  $\Omega$  and field resistance of 250  $\Omega$ . When the machine runs as motor, input to it at no-load is 4 A at 250 V. Calculate the speed and efficiency of the machine when it runs as a motor taking 40 A at 250 V. Armature reaction weakens the field by 4 %. <u>Solution:</u>

Now, when running as a generator, the machine gives 250 V at 1000 r.p.m. If this machine was running as motor at 1000 r.p.m., it will, obviously, have a back emf of 250 V produced in its armature. Hence N1 = 1000 r.p.m. and  $E_{a1}$ = 250 V. When it runs as a motor, drawing 40 A, the back e.m.f. induced in its armature is

$$E_{a} = V_{f} - I_{a}R_{a}$$

$$I_{a} = I_{L} - I_{f}, \quad I_{f} = \frac{250}{250} = 1 \text{ A}$$

$$E_{a2} = 250 - (40 - 1)x \ 0.5 = 230.5 \text{ V}$$

$$\frac{E_{a2}}{E_{a1}} = \frac{k \phi_{2} x N_{2} x \frac{2\Pi}{60}}{k \phi_{1} x N_{f} x \frac{2\Pi}{60}} = \frac{0.96 \phi_{1} x N_{2}}{\phi_{1} x N_{1}} = \frac{0.96 x N_{2}}{N_{1}}$$

$$N_{2} = \frac{E_{a2}}{E_{a1}} x \frac{N_{1}}{0.96} = \frac{230.5}{250} x \frac{1000}{0.96} = 960.4167 \text{ r.p.m.}$$

To calculate the efficiency, the following must be estimated Mechanical and magnetic losses (from no-load can be calculated) Total copper losses

Input power at given load

Output power

At no-load the input power is

 $P_{in} = V_{t}I_{L} = 250x4 = 1000$  wat

The mechanical and magnetic losses =  $P_{in}$  - total copper losses

The mechanical and magnetic losses =

$$P_{in} - I_a^2 R_a - I_f^2 R_f = 1000 - 3^2 x \, 0.5 - 1^2 x \, 250 = 745.5$$
 wat

The copper losses = armature copper loss + field copper loss

The copper losses = 
$$I_a^2 R_a^+ I_f^2 R_f^- = 39^2 x \, 0.5 + 1^2 x \, 250 = 1010.5$$
 wat

At given-load the input power is

 $P_{in} = V_{I} I_{L} = 250x \, 40 = 10000 \text{ wat}$ 

At given-load the total power losses is

The total power losses = The mechanical and magnetic losses + The copper losses

The total power losses = 745.5 + 1010.5 = 1756 wat

At given-load the output power is

The output power = input power - total power losses

The output power = 10000 -1756=8244 wat

Efficiency = 
$$\frac{P_{out}}{P_{in}} x 100 = \frac{8244}{10000} x 100 = 82.44\%$$

# 4-5 Reversing of the DC motors:

To reverse the direction of the motion for the DC motors, this occurs by changing polarities any one from the following

- 1. Changing the polarities of the armature windings.
- 2. Changing the polarities of the field windings.

### 4-6 Starting of the DC motors:

To understanding why need tools to startup the DC motor flows the next, Basic operational voltage equation of a DC motor is given as  $V_{t} = E_{a} + I_{a}R_{a}$  and hence,  $I_{a} = \frac{V_{t} - E_{a}}{R_{a}}$  Now, when the motor is at rest, obviously, the back emf  $E_{a} = 0$ . Hence, armature current at the moment of starting can be given as  $I_{a} = \frac{V_{t}}{R_{a}}$ . In practical DC machines, armature resistance is basically very low, generally about 0.5  $\Omega$ . Therefore, a large current flows through the armature during starting. This current is large enough to damage the armature circuit. Due to this excessive starting current.

- 1. the fuses may blow out and the armature winding and/or commutator brush arrangement may get damaged.
- 2. very high starting torque will be produced (as torque is directly proportional to the armature current), and this high starting torque may cause huge centrifugal force which may throw off the armature winding.
- 3. other loads connected to the same source may experience a dip in the terminal voltage.

A large DC motor will pick up speed rather slowly due to its large rotor inertia. Hence, building up the back emf slowly causing the level of high starting current maintained for quite some time. This may cause severe damage. To avoid this, a suitable DC motor starter must be used. Very small dc motors, however, may be started directly by connecting them to the supply with the help of a contactor or a switch. It does not result in any harm because they gather speed quickly due to small rotor inertia. In this case, the large starting current will die down quickly because of the fast rise in the back emf. To avoid the above dangers while starting a DC motor, it is necessary to limit the starting current. So, a DC motor is started by using a starter. There are various types of dc motor starters, such as three point starters, four point starters, no-load release coil starter, thyristor controller starter etc. The basic concept behind every DC motor starter is adding external resistance to the armature winding during starting. From the followings, three point starters and four point starters are used for starting shunt wound motors and compound wound-motors.

#### <u>4-6-1 Three points starter:</u>

The internal wiring of a three points starter is as shown in the Fig. 4-23. When the connected dc motor is to be started, the lever is turned gradually to the right. When the lever touches point 1, the field winding gets directly connected across the supply, and the armature winding gets connected with resistances R1 to R5 in series. During starting, full resistance is added in series with the armature winding. Then, as the lever is moved further, the resistance is gradually is cut out from the armature circuit. Now, as the lever reaches to position 6, all the resistance is cut out from the armature circuit and armature gets directly connected across the supply. The electromagnet 'E' (no voltage coil) holds the lever at this position. This electromagnet releases the lever when there is no (or low) supply voltage. It can be seen that, when the arm is moved from the position 1 to the last position, the starter resistance gets added in series with the field winding. But, as the value of starter resistance is very small as compared to the shunt resistance, the decrease in shunt field current may be negligible. However, to overcome this drawback a brass or copper arc may be employed within a Three points starter which makes a connection between the moving arm and the field winding, as shown in the figure of four point starters below. When the motor is overloaded beyond a predefined

value, 'overcurrent release electromagnet' D gets activated, which shortcircuits electromagnet E and, hence, releases the lever and the motor is turned off.



Fig. 4-23 Three points starter DC motor

## 4-6-2 Four Point Starters:

Four point starters DC motor can be seen in Fig. 4-24. The main difference between a three point starters and a four point starters is that the no voltage coil (electromagnet E) is not connected in series with the field coil. The field winding gets directly connected to the supply, as the lever moves touching the brass arc (the arc below the resistance studs). The no voltage coil (or Hold-on coil) is connected with a current limiting resistance Rh. This arrangement ensures that any change of current in the shunt field does not affect the current through hold-on coil at all. This means, electromagnetic pull of the hold-on coil will always be sufficient so that the spring does not unnecessarily restore the lever to the off position. A four point starters is used where field current is to be adjusted by means of a field rheostat for the purpose of operating the motor above rated speed by reducing the field current.



Fig. 4-24 Four point starters DC motor

4-6-3 DC series motor starter:

Construction of DC series motor starters is very basic as shown in the Fig.4-25. The start arm is simply moved towards right to start the motor. Thus, maximum resistance is connected in series with the armature during starting and then gradually decreased as the start arm moves towards right. This starter is sometimes also called as a two point starters. The no load release coil holds the start arm to the run position and leaves it when the voltage is lost.



Fig. 4-25 DC series motor starter

## 4-7 Braking of the DC motors:

A running motor may be brought to rest quickly by either mechanical braking or electrical braking. The mechanical braking is applied by means of mechanical break shoes. Hence the smoothness of mechanical braking is dependent on the surface and physical condition of brakes. Smooth braking of a motor can be achieved by electric braking. In many cases electric braking makes more brake power available to the braking process where mechanical brakes are applied. This reduces the wear and tear of the mechanical brakes and reduces the frequency of the replacement of these parts. The electric braking may be done for various reasons such as those mentioned below:

1. To augment the brake power of the mechanical brakes.

2. To save the life of the mechanical brakes.

3. To regenerate the electrical power and improve the energy efficiency.

4. In the case of emergencies to step the machine instantly.

5. To improve the through put in many production process by reducing the stopping time.

The electric braking of a DC motor is of three types, (i) Rheostatic or dynamic braking, (ii) Plugging or reverse current braking and (iii) Regenerative breaking.

#### <u>4-7-1 Rheostatic or dynamic braking:</u>

In case of DC shunt motors, armature is disconnected from the supply and a rheostat (variable resistor) is connected across it. This is done by changing the switch from position 1 to 2 as shown in Fig. 4-26a. The field winding is left connected across the supply. Obviously, now armature is driven by the inertia and hence machine starts acting as a generator. Thus, the machine will now feed the current to the connected rheostat and heat will dissipate at the rate of  $I^2R_{DB}$ . Braking effect is controlled by varying the resistance connected across the armature. Due to the rotation of the armature during motoring mode and due to the inertia, the armature continues to rotate. An emf is induced due to the presence of the field and the rotation. This voltage drives a current through the braking resistance. The direction of this current is opposite to the one which was flowing before change in the connection. Therefore, torque developed also gets reversed. The machine acts like a brake. The torque speed characteristics separate by excited shunt of the machine under dynamic braking mode is as shown in Fig. 4-26b for a particular value of  $R_{DB}$ . The positive torque corresponds to the motoring operation.

In case of DC series motor the excitation current becomes zero as soon as the armature is disconnected from the mains and hence the induced emf also vanishes. In order to achieve dynamic braking the series field must be isolated and connected to a low voltage high current source to provide the field.

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Rather, the motor is made to work like a separately excited machine. When several machines are available at any spot, as in railway locomotives, dynamic braking is feasible. Series connection of all the series fields with parallel connection of all the armatures connected across a single dynamic braking resistor is used in that case.





### <u>4-7-2 Plugging or Reverse current braking:</u>

In this method, armature connections are reversed and hence motor tends to run in opposite direction. Due to reversal of the armature terminals, applied voltage  $V_{t}$  and back emf  $E_{a}$  starts acting in the same direction and hence the total armature current exceeds. To limit this armature current a variable resistor is connected across the armature. This is similar for both series and shunt wound methods. Plugging gives greater braking torque as compared

to rheostatic braking. This method is generally used in controlling elevators, machine tools, printing presses etc.

### 4-7-3 Regenerative braking:

Regenerative braking is used where, load on the motor has very high inertia (e.g in electric trains). When applied voltage to the motor is reduced to less than back emf  $E_a$ , obviously armature current Ia will get reversed, and hence armature torque is reversed. Thus speed falls. As generated emf is greater than applied voltage (machine is acting as a DC generator), power will be returned to the line, this action is called as regeneration. Speed keeps falling, back emf  $E_a$  also falls until it becomes lower than applied voltage and direction of armature current again becomes opposite to  $E_a$ .



Fig. 4-27 plugging

### SINGLE PHASE TRANSFORMER

A transformer is an electrical device that transfers electrical energy between two or more circuits i.e. primary windings (windings connected to power supply) and secondary windings (windings connected to load) without any direct electrical connection without changing its frequency but may be in different voltage level through electromagnetic induction. Electromagnetic induction produces an electromotive force within a conductor which is exposed to time varying magnetic fields. Transformers are used to increase or decrease the alternating voltages in electric power applications. A varying current in the transformer's primary winding creates a varying magnetic flux in the transformer core and a varying field impinging on the transformer's secondary winding. This varying magnetic field at the secondary winding induces a varying electromotive force emf or voltage in the secondary winding due to electromagnetic induction. Making use of Faraday's Law in conjunction with high magnetic permeability core properties, transformers can be designed to efficiently change AC voltages from one voltage level to another within power networks.

### 5-1 Transformer Important:

Electrical power generated in a low voltage level is very cost effective. If the voltage level is increased, the power current is reduced which reduces the ohmic loss, cross-sectional area of the conductor and the capital cost of the system, while improving the voltage regulation. Thus, low level power must be stepped up for efficient electrical power transmission by a step-up transformer at the sending side of the power system network. Similarly, because high voltage cannot be distributed to the consumers directly, it must be stepped down at the receiving end by a step-down transformer. Where the

ratio between high voltage and less voltage is less than two, auto transformers are used; and where the ratio is greater than two, two-winding transformers are used. It can be used to prevent DC from passing from one circuit to the other. it can isolate two circuits electrically.

## 5-2 Transformer Construction:

The construction of transformers varies greatly, depending on their applications, winding voltage and current ratings and operating frequencies. The types of the transformers will show in the following section. Here the focus is on power transformers only. The transformer mainly consists of two windings coupled through a magnetic medium. The magnetic medium is a core. The core is a path for the magnetic flux. The two windings are called primary windings and secondary windings. The primary windings are also called primary coil, which receives energy from the ac source. The secondary windings are also called secondary coil, which receives energy from the primary winding and delivers it to the load. In some transformers, there are an enclosure, which protects the transformer from dirt, moisture, and mechanical damage. Here more common constructional aspects alone are discussed. These can be broadly divided into

- 1. Core construction
- 2. Winding arrangements
- 3. Insulation
- 4. Cooling aspects

# 5-2-1 Core construction:

Transformer core for the power frequency application is made of highly permeable material. The high value of permeability helps to give a low reluctance for the path of the flux and the flux lines mostly confine themselves to the iron. Relative permeability  $\mu$ r well over 1000 are achieved by the

present day materials. Silicon steel in the form of thin laminations is used for the core material. Over the years progressively better magnetic properties are obtained by going in for Hot rolled non-oriented to Hot rolled grain oriented steel. Later better laminations in the form of cold Rolled Grain Oriented (CRGO), -High B (HiB) grades became available. The thickness of the laminations progressively got reduced from over 0.5mm to the present 0.25mm per lamination. These laminations are coated with a thin layer of insulating varnish, oxide or phosphate. The magnetic material is required to have a high permeability  $\mu$  and a high saturation flux density, a very low remanence Br and a small area under the B-H loop-to permit high flux density of operation with low magnetizing current and low hysteresis loss. The resistivity of the iron sheet itself is required to be high to reduce the eddy current losses. The eddy current itself is highly reduced by making the laminations very thin. If the lamination is made too thin then the production cost of steel laminations increases. The steel should not have residual mechanical stresses which reduce their magnetic properties and hence must be annealed after cutting and stacking. In the case of very small transformers (from a few volt-amperes to a few kilo volt-amperes) hot rolled silicon steel laminations in the form of E & I, C & I or O as shown in Fig. 5-1 are used and the core cross section would be a square or a rectangle. The percentage of silicon in the steel is about 3.5. above this value the steel becomes very brittle and also very hard to cut. The saturation flux density of the present day steel lamination is about 2 Tesla. Broadly classifying, the core construction can be separated into core type and shell type.

<u>5-2-1-1 Core type construction:</u>

In a core type construction the winding surrounds the core. A few examples of single phase and three phase core type constructions are shown in Fig. 5-2.



Fig. 5-1 E and I,C and I and O Type Laminations



(Core Type Transformer Cruciform Section

Fig. 5-2 Core type transformer

In core type transformer, the magnetic core is built of laminations to form a rectangular frame and the windings are arranged concentrically with each other around the legs or limbs. The top and bottom horizontal portion of the core are called yoke. The yokes connect the two limbs and have a cross sectional area equal to or greater than that of limbs. Each limb carries one half of primary and secondary. The two windings are closely coupled together to reduce the leakage reactance. The low voltage winding is wound near the core and high voltage winding is wound over low voltage winding away from core in order to reduce the amount of insulating materials required. The cylindrical coils will have different layers and each layer will be insulated from the other with the help of materials like paper, cloth, micarta board. This type of transformer has some advantages as, maintenance and repair are easy. Ventilation and Colling transformer windings are easy. Less leakage flux and high efficiency.

### 5-2-1-2 Shell type construction:

In the shell-type construction the iron almost entirely surrounds the copper and i.e. the windings are put around the central limb and the flux path is completed through two side limbs. The central limb carries total mutual flux while the side limbs forming a part of a parallel magnetic circuit carry half the total flux. The cross sectional area of the central limb is twice that of each side limbs. The coils are form-wound but are multi layer disc type usually wound in the form of pancakes. Paper is used to insulate the different layers of the multi-layer discs. The whole winding consists of discs stacked with insulation spaces between the coils. These insulation spaces form the horizontal cooling and insulating ducts. Such a transformer may have the shape of a simple rectangle or may also have a distributed form. to reduce the amount the highvoltage insulation required, the low-voltage coils are placed adjacent to the core.



Shell Type Transformers Distributed Form

Fig. 5-2 Shell type transformer

# 5-2-2 Transformer windings arrangement:

Normally the two winding transformer there will be two windings; one is the L V winding, another is the H V winding. The windings are made up of aluminum or copper conductors either round or square cross-section. Square cross-section conductors can be made up of solid conductor or multiple strips

connected in parallel. The L V windings are usually of two types; one is the cylindrical or helix. The HV winding can also be of two type either cylindrical or crossover. The next important constructional feature is the insulator. Insulators are used in different places of a transformer, and they are classified as major insulation which are between let us say core and the L V winding, L V winding and the H V winding, and the minor insulation which are usually between the turns of the same winding or the layers of the same winding. Now there are different types of insulation materials that are used. For example, pressboard cylinders, then paper, transformer oil at different purpose, different insulations are used. The insulation material are characterized by the maximum temperature at which they can maintain their insulation properties, and this is the weakest constructional component of a transformer, and more often not the quality of the insulator will determine the reliability or even the size of a transformer. Other than these three main constructional elements, there are many other elements that are used; for example, the bushings. The bushings are these protruding elements through which external connection is made to the transformer. These are usually porcelain cylinders hollow porcelain cylinders which these sets through which the connecting conductors are placed. They can be for high voltage, they can be oil field, or they can be of capacitor type. So, bushing is provided for external connection and then cooling arrangement. For small transformer separate cooling arrangements are not provided. The cooling is provided by the tank itself through which the transformer is placed. For large transformer they are usually immersed in transformer oil so that heat can be extracted from the inner parts of the transformer that is the core, and the L V winding which is inside the H V winding. Transformer coils can be broadly classified in to concentric coils and sandwiched coils depending upon the position of the coil inside the transformer. These coil have three most common types of coils viz. helical, cross over and disc coils.

## 5-2-2 -1 Helical Windings:

One very common cylindrical coil arrangement is the helical winding. This is made up of large cross section rectangular conductor wound on its flat side. The coil progresses as a helix. This is commonly used for LV windings. The insulation requirement also is not too high. Between layers no insulation (other than conductor insulation) is needed as the voltage between layers is low. The complexity of this type of winding rapidly increases as the current to be handled becomes more. The conductor cross section becomes too large and difficult to: handle. The eddy current losses in the conductor rapidly increases. Hence two or more conductors have to be wound and connected in parallel. The parallel circuits bring in problems of current sharing between the circuits. Transpositions of the parallel paths have to be adopted to reduce unequal current distribution. The modern practice is to use continuously transposed and bunched conductors.

# 5-2-2 -2 Cross over coils:

The second popular winding type is the cross over coil. These are made of circular conductors not exceeding 5 to 6 sq mm in cross section. These are used for HV windings of relatively small transformers. These turns are wound in several layers. The length and thickness of each block is made in line with cooling requirements. A number of such blocks can be connected in series, leaving cooling ducts in between the blocks, as required by total voltage requirement.

# 5-2-2 -3 Disc coils:

Disc coils consist of flat conductors wound in a spiral form at the same place spiralling outwards. Alternate discs are made to spiral from outside towards the center. Sectional discs or continuous discs may be used. These have excellent thermal properties and the behavior of the winding is highly predictable. Winding of a continuous disc winding needs specialized skills.

### 5-2-2-4 Sandwich coils:

Sandwich windings are more common with shell type core construction. They permit easy control over the short circuit impedance of the transformer. By bringing HV and LV coils close on the same magnetic axis the leakage is reduced and the mutual flux is increased. By increasing the number of sandwiched coils the reactance can be substantially reduced.

### 5-2-3 Insulation:

The insulation used in the case of electrical conductors in a transformer is varnish or enamel in dry type of transformers. In larger transformers to improve the heat transfer characteristics the conductors are insulated using un-impregnated paper or cloth and the whole core-winding assembly is immersed in a tank containing transformer oil. The transformer oil thus has dual role. It is an insulator and also a coolant. The porous insulation around the conductor helps the oil to reach the conductor surface and extract the heat. The conductor insulation may be called the minor insulation as the voltage required to be with stood is not high. The major insulation is between the windings. Annular bakelite cylinders serve this purpose. Oil ducts are also used as part of insulation between windings. The oil used in the transformer tank should be free from moisture or other contamination to be of any use as an insulator.

### 5-2-4 Cooling aspects:

The main source of heat generation in transformer is its copper loss or I<sup>2</sup>R loss. Although there are other factors contribute heat in transformer such as hysteresis and eddy current losses but contribution of I<sup>2</sup>R loss dominate them. If this heat is not dissipated properly, the temperature of the transformer will rise continually which may cause damages in paper insulation and liquid insulation medium of transformer. So it is essential to control the temperature with in permissible limit to ensure the long life of transformer by reducing thermal degradation of its insulation system. In electrical power transformer we use external transformer cooling system to accelerate the dissipation rate of heat of transformer. There are different transformer cooling methods available for trans former, as air natural (AN), air forced (AF) or Air Blast, oil natural air natural (ONAN), oil natural air forced (ONAF), oil forced air forced (OFAF), oil natural water forced (ONAF) and oil forced water forced (OFWF). These methods of cooling are illustrated in the next chapter in some details.

### <u>5-3 Transformer – Working Principle:</u>

Fig. 5-3 shows a schematic diagram of a single-phase transformer. There are two types of windings in a single-phase transformer. These are called primary and secondary windings or coils. The primary winding is connected to the alternating voltage source and the secondary winding is connected to the load. The primary and secondary winding parameters are represented by the suffix p or 1 and s or 2, respectively. A sinusoidal current flows in the primary winding when it is connected to an alternating voltage source. This current establish a flux u which moves from the primary winding to the secondary winding through low reluctance magnetic core. About 95 % of this flux moves from the primary to the secondary through the low reluctance path of the

magnetic core and this flux is linked by the both windings and a small percent of this flux links to the primary winding. According to the Faradays laws of electromagnetic induction, a voltage will be induced across the secondary winding as well as in the primary winding. Due to this voltage, a current will flow through the load if it is connected with the secondary winding. Hence, the primary voltage is transferred to the secondary winding without a change in frequency.



Fig. 5-3 Schematic diagram of a single-phase transformer

#### 5-4 Flux in Transformer:

The current in the primary winding establishes a flux. The flux that moves from primary to secondary and links both the windings is called the mutual flux and its maximum value is represented by  $\phi_m$ . Flux which links only the primary winding and completes the magnetic path through the surrounding air is known primary leakage flux. The primary leakage flux is denoted by  $\phi_{11}$ . Similarly, secondary leakage flux is that flux which links only the secondary winding and completes the magnetic path through the surrounding air. The secondary leakage flux is denoted by  $\phi_{21}$ . Mutual and leakage fluxes are shown in Fig. 5-4.



Fig. 5-4 Flux types in the transformer

# 5-4 E.M.F Equation of a Transformer:

The primary winding draws a current when it is connected to an alternating voltage source. This primary sinusoidal current produces a sinusoidal flux  $\phi$  that can be expressed as,

$$\phi = \phi_{\max} \sin \omega t \qquad 5.1$$

Instantaneous emf induced in the primary winding is,

$$e_{1} = -N_{1} \frac{d\phi}{dt} \qquad 5.2$$

By substituting from eq. 5.1 into eq. 5.2 it is found that, instantaneous emf induced in the primary winding is,

$$e_{1} = -N_{1} \frac{d(\phi_{\max} \sin \omega t)}{dx} = -\omega N_{1} \phi_{\max} \cos \omega t$$
  

$$e_{1} = \omega N_{1} \phi_{\max} \sin(\omega t - 90^{\circ})$$
5.3

Let  $\omega N_{1} \phi_{max}$  is equal maximum induced voltage which represented by  $E_{m1}$ By substituting about the maximum induced voltage in eq. 5.3 it is found that, instantaneous emf induced in the primary winding is,

$$e_1 = E_{m1} \sin(\omega t - 90^\circ) \qquad 5.4$$

The rms value of the primary emf is,

$$E_{1} = \frac{E_{m1}}{\sqrt{2}} = \frac{\omega N_{1} \phi_{max}}{\sqrt{2}} = \frac{2\Pi f N_{1} \phi_{max}}{\sqrt{2}}$$

$$E_{1} = 4.44f N_{1} \phi_{\text{max}}$$
 5.5

Similarly, instantaneous emf induced in the secondary winding is,

$$e_2 = -N_2 \frac{d\phi}{dt}$$
 5.6

By similar manner it is found that, the rms value of the secondary emf is,

$$E_{2} = 4.44 f N_{2} \phi_{\text{max}}$$
 5.7

The primary and secondary voltages can be determined from eqs. 5-6 and 5-7 if other parameters are known.

#### 5-5 Ideal Transformer:

An ideal transformer is one which does not supply any energy to the load i.e., the secondary winding is open circuited. The main points of an ideal transformer are (i) no winding resistance, (ii) no leakage flux and leakage inductance, (iii) self-inductance and mutual inductance are zero, (iv) no losses due to resistance, inductance, hysteresis or eddy current and (v) coefficient of coupling is unity. Fig. 5-5a shows an ideal transformer where the secondary winding is left open. A small magnetizing current  $I_m$  will flow in the primary winding when it is connected to the alternating voltage source,  $V_{\perp}$ . This magnetizing current lags behind the supply voltage,  $V_{\perp}$  by 90° and produces the flux  $\phi$ , which induces the primary and secondary emfs. These emfs lag behind the flux,  $\phi$  by 90°. The magnitude of primary induced emf  $E_{\perp}$  and supply voltage  $V_{\perp}$  is the same, but are 180° out of phase as shown in Fig. 5-5b.

# 5-6 Turns ratio for Transformer:

Turns ratio is an important parameter for drawing an equivalent circuit of a transformer. The turns ratio is used to identify the step-up and step-down transformers. By dividing eq. 5-1 on eq. 5-6 it is found that,



Fig. 5-5 Ideal transformer and phasor diagram

 $\frac{e_{1}}{e_{2}} = \frac{N_{1}}{N_{1}}$  5.8

Assume that  $\frac{N_{1}}{N_{1}} = a$  it will call turns ratio so the eq. 5.8 becomes

$$\frac{e_1}{e_2} = \frac{N_1}{N_1} = a$$
 5.9

By similar manner when divided eq. 5-5 on 5-7 it is found that,

$$\frac{E_{1}}{E_{2}} = \frac{N_{1}}{N_{1}} = a \qquad 5.10$$

In case of N2 > N1, the transformer is called a step-up transformer. Whereas for N1 > N2, the transformer is called a step-down transformer. The losses are

zero in an ideal transformer. In this case, the input power of the transformer is equal to its output power and this yields,

$$V_{1}I_{1} = V_{2}I_{2}$$
 5.11

From the above eq. it is found that,

$$\frac{V_{1}}{V_{2}} = \frac{I_{2}}{I_{1}} = a$$
 5.12

The input and output power of an ideal transformer is,

$$P_{in} = V_{1}I_{1}\cos\phi_{1}$$

$$P_{out} = V_{2}I_{2}\cos\phi_{2}$$
5.13

For an ideal condition, the angle  $\phi_1$  is equal to the angle  $\phi_2$  and the output power can be re-arranged as,

$$\boldsymbol{P}_{out} = \frac{V_{1}}{a} a \boldsymbol{I}_{1} \cos \boldsymbol{\theta}_{1} = V_{1} \boldsymbol{I}_{1} \cos \boldsymbol{\theta}_{1} = \boldsymbol{P}_{in} \quad 5.14$$

From eq. 5-14, it is seen that the input and the output power are the same in case of an ideal transformer. Similarly, the input and output reactive powers are,

$$Q_{in} = V_1 I_1 \sin \theta_1 = V_2 I_2 \sin \theta_2 = Q_{out} \quad 5.15$$

From eq. 5-14and eq. 5-15, the input and output power and reactive power can be calculated if other parameters are given.

For developing equivalent circuit of a transformer, it is necessary to refer the parameters from the primary to the secondary or the secondary to the primary. These parameters are resistance, reactance, impedance, current and voltage. The ratio of primary voltage to secondary voltage and ratio of primary current to secondary current are calculated as eq. 5-12 now the impedance ratio is calculated

$$\frac{V_{-1}}{V_{-2}} = \frac{a}{1} = a^{2}, \frac{V_{-1}}{V_{-2}} = a^{2}$$
5.16  
But  $\frac{V_{-1}}{I_{-1}} = Z_{-1}$  and  $\frac{V_{-2}}{I_{-2}} = Z_{-2}$  by substituting into eq. 5.16 it is found that  
 $\frac{Z_{-1}}{Z_{-2}} = a^{2}$ 
5.17

From eq. 5-17, it can be concluded that the impedance ratio is equal to the square of the turns ratio. The important points for transferring parameters are (i)  $R_1$  in the primary becomes  $\frac{R_1}{a^2}$  when referred to the secondary, (ii)  $R_2$  in the secondary becomes  $a^2R_2$  when referred to the primary, (iii)  $X_1$  in the primary becomes  $\frac{X_1}{a^2}$  when referred to the secondary, and (iv)  $X_2$  in the secondary becomes  $a^2X_2$  when referred to the primary.

### Problem 5-1:

The number of turns in the secondary coil of a 22 kVA, 2200 V/220 V singlephase transformer is 50. The transformer is connected to a 2200 V, 50 Hz source. The transformer is connected to a 220 V, 50 Hz source. Neglect all kinds of losses in the transformer. Find the following:

- (i) number of primary turns.
- (ii) primary full load current.
- (iii) secondary full load current.
- (iv) load impedance referred to the primary.
- (v) the mutual flux in the core.

### Solution:
To find the number of primary turns

$$\frac{N_{1}}{N_{2}} = \frac{V_{1}}{V_{2}}, N_{1} = \frac{V_{1}}{V_{2}}N_{2} = \frac{2200}{220}x50 = 500 \text{ turns}$$

To calculate the primary full load current

$$I_{1} = \frac{22000}{2200} = 10$$
 A

To calculate the secondary full load current

$$I_{2} = \frac{22000}{220} = 100$$
 A

To calculate load impedance referred to the primary

$$\frac{Z_{1}}{Z_{2}} = (\frac{V_{1}}{V_{2}})^{2}, Z_{1} = (\frac{V_{1}}{V_{2}})^{2} Z_{2} = (\frac{2200}{220})^{2} x 4 = 400 \text{ ohm}$$

To calculate the mutual flux in the core

$$V_{1} = 4.44f N_{1} \phi_{\text{max}}$$
  
 $\phi_{\text{max}} = \frac{V_{1}}{4.44f N_{1}} = \frac{2200}{4.44x \, 50x \, 500} = 0.02$  weber

#### 5-7 Equivalent Circuit of a Transformer:

Windings of a transformer are not connected electrically. The windings are magnetically coupled with each other. In this case, it is tedious to do proper analysis. Therefore, for easy computation and visualization, the practical transformer needs to be converted into an equivalent circuit by maintaining same properties of the main transformer. In the equivalent circuit, the related parameters need to be transferred either from the primary to the secondary or vice versa. This means that, In a practical transformer (a) Some leakage flux is present at both primary and secondary sides. This leakage gives rise to leakage reactances at both sides, which are denoted as  $X_1$  and  $X_2$  respectively. (b) Both the primary and secondary winding possesses resistance, denoted as  $R_1$  and  $R_2$  respectively. These resistances causes voltage drop as,  $I_1R_1$  and  $I_2R_2$  and also copper loss  $I_1^2R_1$  and  $I_2^2R_2$ . (c) Permeability of the core can not be

infinite, hence some magnetizing current is needed. Mutual flux also causes core loss in iron parts of the transformer. We need to consider all the above things to derive equivalent circuit of a transformer. Resistances and reactances of transformer, which are described above, can be imagined separately from the windings as shown in the Fig. 5-6. Hence, the function of windings, thereafter, will only be the transforming the voltage.



Fig. 5-6 Practical single phase transformer

At no-load, when an alternating source is applied in the primary, the source will supply the current for magnetizing the core of transformer. But this current is not the actual magnetizing current, it is little bit greater than actual magnetizing current. Actually, total current supplied from the source has two components, one is magnetizing current which is merely utilized for magnetizing the core and other component of the source current is consumed for compensating the core losses in transformer. Because of this core loss component, the source current is not exactly at 90° lags of supply voltage, but it lags behind an angle  $\theta$  is less than 90°. If total current supplied from source is  $I_0$ , it will have one component in phase with supply voltage  $V_1$  and this component of the current  $I_w$  is core loss component. This component is taken in phase with source voltage, because it is associated with active or

working losses in transformer. Other component of the source current is denoted as  $I_m$ . This component produces the alternating magnetic flux in the core, so it is watt-less; means it is reactive part of the transformer source current. Hence  $I_m$  will be in quadrature with  $V_1$  and in phase with alternating flux  $\phi$ . The no load current  $I_0$  is divided into, pure inductance  $X_0$  (taking magnetizing components  $I_m$ ) and non induction resistance  $R_0$  (taking working component  $I_w$  ) which are connected into parallel across the primary. The value of E<sub>1</sub> can be obtained by subtracting  $I_1(R_1+JX_1)$  from  $V_1$ . The value of  $R_0$  and  $X_0$  can be calculated as,  $R_0 = \frac{E_1}{I}$  and  $X_0 = \frac{E_1}{I}$ . But, using this equivalent circuit does not simplifies the calculations. To make calculations simpler, it is preferable to transfer current, voltage and impedance either to primary side or to the secondary side. In that case, we would have to work with only one winding which is more convenient. Fig. 5-7 shows an exact equivalent circuit referred to the primary where all the parameters are transferred from the secondary to the primary and these parameters are,



Fig. 5-7 Exact equivalent circuit of transformer referred to primary

Fig. 5-8 shows the exact equivalent circuit referred to the secondary where all the parameters are transferred from the primary to the secondary. These



Fig. 5-8 Exact equivalent circuit of transformer referred to secondary <u>5-8 Approximate Equivalent Circuit:</u>

The no-load current is very small as compared to the rated primary current. Therefore, there is a negligible voltage drop due to  $R_1$  and  $X_1$ . In this condition, it can be assumed that the voltage drop across the no-load circuit is the same as the applied voltage without any significant error. The approximate equivalent circuit can be drawn by shifting the no-load circuit across the supply voltage,  $V_1$ . Fig. 5-9 shows an approximate equivalent circuit referred to the primary. The total resistance, reactance and impedance referred to the primary are,

$$R_{01} = R_1 + R_2' = R_1 + a^2 R_2,$$
  
$$X_{01} = X_1 + X_2' = X_1 + a^2 X_2,$$

The no-load circuit resistance and reactance are,



Fig. 5-9 Approximate equivalent circuit referred to the primary Fig. 5-10 shows an approximate equivalent circuit referred to the secondary. The total resistance, reactance and impedance referred to the secondary is,



Fig. 5-10 Approximate equivalent circuit referred to the secondary

$$R_{02} = R_{2} + R_{1} = R_{2} + \frac{R_{1}}{a^{2}},$$
  

$$X_{02} = X_{2} + X_{1} = X_{2} + \frac{X_{1}}{a^{2}},$$
  

$$Z_{02} = R_{02} + jX_{02},$$

The no-load circuit resistance and reactance referred to the secondary are,

$$R_{0}' = \frac{V_{1}'}{I_{w}}, X_{0}' = \frac{V_{1}'}{I_{m}}$$

# Problem 5-2

A 2.5 kVA, 200 V/40 V single-phase transformer has the primary resistance and reactance of 3 and 12  $\Omega$ , respectively. On the secondary side, these values are 0.3 and 0.1  $\Omega$ , respectively. Find the equivalent impedance referred to the primary and the secondary.

## Solution:

The value of the turns ratio is,

$$a = \frac{V_1}{V_2} = \frac{200}{40} = 5$$

The total resistance, reactance and impedance referred to the primary can be determined as,

$$R_{01} = R_1 + a^2 R_2 = 3 + 25x \ 0.3 = 10.5\Omega,$$
  
$$X_{01} = X_1 + a^2 X_2 = 12 + 25x \ 0.1 = 14.5\Omega,$$
  
$$Z_{01} = R_{01} + j X_{01} = \sqrt{10.5^2 + 14.5^2} = 17.9\Omega$$

The total resistance, reactance and impedance referred to the secondary are calculated as,

$$R_{02} = R_2 + \frac{R_1}{a^2} = 0.3 + \frac{3}{25} = 0.42\Omega,$$
  

$$X_{02} = X_2 + \frac{X_1}{a^2} = 0.1 + \frac{12}{25} = 0.58\Omega,$$
  

$$Z_{02} = R_{02} + j X_{02} = \sqrt{0.42^2 + 0.58^2} = 0.72,$$
  
5-9 Phasor diagram of a Transformer:

The phasor diagram for the transformer on load depends on the nature of the load power factor. If the load power factor is lagging  $\cos \Phi_2$ , the complete phasor diagram is shown in the Fig. 5-11. Steps to draw the phasor diagram are,



Fig. 5-11 The phasor diagram for the transformer on load condition

- 1. Consider flux  $\phi$  as reference
- 2.  $E_1$  lags  $\phi$  by 90°. Reverse  $E_1$  to get  $E_1$ .
- 3.  $E_1$  and  $E_2$  are inphase
- 4. Assume  $V_{2}$  in a particular direction
- 5. the current  $I_2$  lags  $V_2$  by angle  $\Phi_2$ .
- 6. Add  $I_2R_2$  and  $I_2X_2$  to to get  $E_2$ .
- 7. Reverse  $I_2$  to get  $I_2'$ .
- 8. Add  $I_0$  and  $I_2$ ' to get  $I_1$ .
- 9. Add  $I_1R_1$  and to  $-E_1$  to get  $V_1$ .

Angle between  $V_1$  and  $I_1$  is  $\Phi_1$  and  $\cos \Phi_1$  is primary power factor. Remember that  $I_1X_1$  leads  $I_1$  direction by 90° and  $I_2X_2$  leads  $I_2$  by 90° as current through inductance lags voltage across inductance by 90°.

### 5-10 Voltage regulation of a Transformer:

The voltage regulation is the percentage of voltage difference between no load and full load voltages of a transformer with respect to its full load voltage. When the Transformer is loaded with a constant supply voltage, the terminal voltage changes depending upon the load and its power factor. The algebraic difference between the no-load and full load terminal voltage is measured in terms of voltage regulation.

The voltage regulation = 
$$\frac{E_2 - V_2}{E_2} \times 100\%$$

where,

 $E_2$  - secondary terminal voltage at no load  $V_2$  - secondary terminal voltage at full load

When all the quantities are referred to the primary side of the transformer, the voltage regulation equation becomes

The voltage regulation =  $\frac{V_{1} - E_{1}}{V_{1}} \times 100\%$ 

where,

 $V_{1}$ Primary – Primary terminal voltage at no load  $E_{1}$ – Primary terminal voltage at full load

# **5-11 Transformer tests:**

To determine the transformer two tests must be done. These testes are opencircuit test and short circuit test These tests are very economical and convenient, because they furnish the required information without actually loading the transformer. In fact, the testing of very large AC machinery consists of running two tests similar to the open and short-circuit tests of a transformer.

### 5-11-1 Open-circuit or No-load Test:

The purpose of this test is to determine no-load loss or core loss and no-load current  $I_0$  which is helpful in finding magnetizing inductance  $X_0$  and core loss resistance  $R_0$ . One winding of the transformer whichever is convenient but usually high voltage winding is left open and the other is connected to its supply of normal voltage and frequency. A wattmeter W, voltmeter V and an ammeter A are connected in the low voltage winding *i.e.* primary winding in the present case. With normal voltage applied to the primary, normal flux will be set up in the core, hence normal iron losses will occur which are recorded

by the wattmeter. As the primary no-load current  $I_0$  (as measured by ammeter) is small (usually 2 to 10% of rated load current), copper loss is negligibly small in primary and nil in secondary (it being open). Hence, the wattmeter reading represents practically the core loss under no-load condition. It should be noted that since  $I_0$  is itself very small, the pressure coils of the wattmeter and the voltmeter are connected such that the current in them does not pass through the current coil of the wattmeter. Sometimes, a high-resistance voltmeter is connected across the secondary. The reading of the voltmeter gives the induced emf. in the secondary winding. This helps to find transformation ratio a. with help of the no-load vector diagram which shown in Fig. 5.12. and if the used the open circuit test which connected as shown in Fig. 5-13., the following can be calculated



Fig. 5-12 No-load phasor diagram

$$P_{0} = V_{1}I_{0}\cos\phi_{0}, \quad \phi_{0} = \cos^{-1}(\frac{P_{0}}{V_{1}I_{0}})$$

$$I_{m} = I_{0}\sin\phi_{0}, \quad I_{w} = I_{0}\cos\phi_{0}$$

$$R_{0} = \frac{V_{1}}{I_{w}}, \quad X_{0} = \frac{V_{1}}{I_{m}}$$



Fig. 5-13 Open circuit test for transformer.

## 5-11-2 Short-circuit or impedance test:

This is an economical method for determining the following:

(*i*) Equivalent impedance  $(Z_1 \text{ or } Z_2)$ , leakage reactance  $(X_1 \text{ or } X_2)$  and total resistance  $(R_1 \text{ or } R_2)$  of the transformer as referred to the winding in which the measuring instruments are placed. (*ii*) Copper losses at full load (and at any desired load). This loss is used in calculating the efficiency of the transformer. (*iii*) Knowing  $Z_1 \text{ or } Z_2$ , the total voltage drop in the transformer as referred to primary or secondary can be calculated and hence regulation of the transformer determined. In this test, one winding, usually the low-voltage winding, is solidly short-circuited by a thick conductor (or through an ammeter which may serve the additional purpose of indicating rated load current. A low voltage usually 5 to 10% of normal primary voltage at correct

frequency is applied to the primary and is cautiously increased till full-load currents are flowing both in primary and secondary Since, in this test, the applied voltage is a small percentage of the normal voltage, the mutual flux produced is also a small percentage of its normal value. Hence, core losses are very small with the result that the wattmeter reading represent the full-load copper loss for the whole transformer *i.e.* both primary copper loss and secondary copper loss. The equivalent circuit of the transformer under short-circuit condition is shown in Fig. 5-14. If  $V_{scl}$  is the voltage required to circulate rated load currents, then the equivalent parameters at this test can be calculated with help reading of the voltammeter  $V_{scl}$ , ammeter  $I_{scl}$ , wattmeter  $P_{sc}$ , where the equivalent resistance for primary and secondary conduct is  $R_{01} = \frac{P_{sc}}{I_{scl}}$  the equivalent impedance for primary and secondary

circuit is  $Z_{sc1} = \frac{V_{sc1}}{I_{sc1}}$  and the equivalent reactance for primary and secondary circuit is  $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$ 

Fig. 5-14 The short circuit test

to find each resistance of primary winding and secondary windings by simply approximation may be divided the equivalent resistance on 2 or by calculating the primary winding through the DC test by impressing small DC voltage in the primary windings about 2% of rated and increasing gradually even the rated current will pass and by dividing the DC voltage on this current the primary resistance  $R_1$  can be calculated and the secondary winding resistance can be calculated as  $R_2' = R_{02} - R_1$  to find each reactance of primary winding and secondary windings by simply approximation may be divided the equivalent reactance on 2 i.e.  $X_1 = X_2' = \frac{X_{02}}{2}$ .

Problem 5-3:

Obtain the equivalent circuit of a 200/400-V, 50-Hz, 1-phase transformer from the following test data :

O.C test : 200 V, 0.7 A, 70 W - on L.V. side

S.C. test : 15 V, 10 A, 85 W – on H.V. side

Calculate the secondary voltage when delivering 5 kW at 0.8 p.f. lagging, the primary voltage being 200V

Solution:

From the open circuit test, the iron core loss resistance and magnetization reactance can be calculated as

$$\phi_0 = \cos^{-1}(\frac{P_0}{V_1 I_0}) = \cos^{-1}(\frac{70}{200x 0.7}) = 60^\circ$$
  
$$I_m = 0.7 \sin 60^\circ = 0.606 \text{A}, \quad I_w = 0.7 \cos 60^\circ = 0.35 \text{A}$$
  
$$R_0 = \frac{200}{0.35} = 571.4\Omega, \quad X_0 = \frac{200}{0.606} = 330\Omega$$

these values refer to primary *i.e.* low-voltage side.

From short circuit test, It may be noted that in this test, instruments have been placed in the secondary *i.e.* high-voltage winding whereas the low-voltage winding *i.e.* primary has been short-circuited.

the equivalent impedance for primary and secondary circuit is

$$Z_{02} = \frac{V_{sc}}{I_{sc2}} = \frac{15}{10} = 1.5\Omega, \quad a = \frac{400}{200} = 2$$
$$Z_{01} = \frac{1.5}{4} = 0.375\Omega$$

$$R_{02} = \frac{85}{100} = 0.85\Omega$$

the equivalent resistance is

$$R_{01} = \frac{R_{02}}{a^2} = \frac{0.85}{4} = 0.21\Omega$$

the equivalent reactance for primary and secondary circuit is  $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{0.375^2 - 0.21^2} = 0.31\Omega$ Output current  $I_2 = \frac{5000}{0.8}x \, 400 = 15.6A$   $X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{1.5^2 - 0.85^2} = 1.24\Omega$ Total transformer drop as referred to secondary  $= I_2(R_{02}\cos\phi_2 + X_{02}\sin\phi_2) = 15.6(0.85x \, 0.8 + 1.24x \, 0.6) = 22.2 V$ 

$$V_{2} = 400 - 22.2 = 377.8V$$

### 5-12 Losses and efficiency in the Transformer:

In a static transformer, there are no friction or windage losses. Hence, the only losses occurring are:

(*i*) core or iron loss: It includes both hysteresis loss and eddy current loss. Because the core flux in a transformer remains practically constant for all loads (its variation being 1 to 3% from no-load to full-load). The core loss is practically the same at all loads. Hysteresis loss  $W_h = \eta B_{\text{max}}^{1.6} f V$  watt; eddy current loss  $W_e = P B_{\text{max}}^2 f^2 t^2$  watt These losses are minimized by using steel of high silicon content for the core and by using very thin laminations. Iron or core loss is found from the *O.C.* test. The input of the transformer when on no-load measures the core loss. (*ii*) Copper loss. This loss is due to the ohmic resistance of the transformer windings. Total copper loss  $=I_1^2 R_{01} + I_2^2 R_{02}$ . It is clear that copper loss is proportional to (current)2 or kVA2. In other words, copper loss at half the full-load is one-fourth of that at full-load. As is the case with other types of electrical machines, the efficiency of a transformer at a particular load and

power factor is defined as the output divided by the input-the two being measured in the same units (either watts or kilowatts).

Efficiency = 
$$\frac{P_{output}}{P_{input}} x 100$$
  
 $P_{input} = V_1 I_1 \cos \phi_1$   
 $P_{output} = P_{input} - Losses$   
Problem 5-4:

A 10 kVA, 1 phase, 50 Hz, 500/250 V transformer gave following test results:

OC test (LV) side : 250 V, 3.0 A, 200 W

SC test (LV) side : 15 V, 30 A, 300 W.

Calculate efficiency at full load, 0.8 p.f. lagging.

## Solution:

For efficiency calculations, full load current should be calculated, on the

L.V. side in this case,

Full load current  $=\frac{10000}{240}=40$ A

Short-circuit test data have been given at 30 A current on the L.V. side. So the copper losses at 40 A L.V. side  $= (\frac{40}{30})^2 x 300 = 533.4$  W At rated voltage, iron losses (from O.C. test) = 200 W

F.L. Output at 0.8 P.F. = 10000x 0.8 = 8000 W

 $\eta = \frac{8000}{8000 + 200 + 533.33} \times 100 = 91.6\%$ 

5-13 Auto Transformer:

Auto transformer is kind of electrical transformer where primary and secondary shares same common single winding. So basically it's a one winding transformer. It is only one winding wound on a laminated core. An auto transformer is similar to a two winding transformer but differ in the way the primary and secondary winding are interrelated. On load condition, a part of the load current is obtained directly from the supply and the remaining part is obtained by transformer action. An Auto transformer works as a voltage regulator. Some of the advantages of auto-transformer are that, it is smaller in size, cheap in cost, low leakage reactance, increased kVA rating and low exciting current. It has some disadvantages as, impedance is low, Because of common neutral in a star / star connected auto transformer it is not possible to earth neutral of one side only. Both their sides should have their neutrality either earth or isolated and It is more difficult to maintain the electromagnetic balance of the winding when voltage adjustment tappings are provided.

#### 5-13-1 Construction of auto transformer:

An auto transformer consists of a single copper wire, which is common in both primary as well as secondary circuit. The copper wire is wound a laminated silicon steel core, with at least three tappings taken out. Secondary and primary circuit share the same neutral point of the winding. The construction is well explained in the diagram. Variable turns ratio at secondary can be obtained by the tappings of the winding as shown in the Fig. 5-15, or by providing a smooth sliding brush over the winding. Primary terminals are fixed. Thus, in an auto transformer, you may say, primary and secondary windings are connected magnetically as well as electrically.



Fig. 5-15 Auto transformer

### 5-13-2 Auto transformer working:

As I have described just above, an auto transformer has only one winding which is shared by both primary and secondary circuit, where number of turns shared by secondary are variable. EMF induced in the winding is proportional to the number of turns. Therefore, the secondary voltage can be varied by just varying secondary number of turns. As winding is common in both circuits, most of the energy is transferred by means of electrical conduction and a small part is transferred through induction. Fig. 5-16 shows both step down and step-up auto-transformers. As shown in Fig. 5-16 (*a*), *AB*, is primary winding having  $N_1$  turns and *BC* is secondary winding having  $N_2$  turns. Neglecting iron losses and no-load current.



Fig. 5-16 Step down step up auto transformer

$$\frac{V_{1}}{V_{2}} = \frac{I_{2}}{I_{1}} = a$$

The current in section *CB* is vector difference of *I*2 and  $I_1$ . But as the two currents are practically in phase opposition, the resultant current is  $(I_2 - I_1)$  where  $I_2$  is greater than  $I_1$ . As compared to an ordinary 2- winding transformer of same output, an auto-transformer has higher efficiency but smaller size. Moreover, its voltage regulation is also superior.

Saving of copper of auto transformer:

Volume and hence weight of copper, is proportional to the length and area of the cross-section of the conductors. Now, length of conductors is proportional to the number of turns and cross-section depends on current. Hence, weight is proportional to the product of the current and number of turns. With reference to Fig. 5-16, weight of copper in section *AC* is  $\alpha (N_1 - N_2) I_1$ ; weight of copper in section *BC* is  $N_2 \alpha (I_2 - I_1)$ . Total weight of copper in auto-transformer  $\alpha (N_1 - N_2) I_1 + N_2 (I_2 - I_1)$ . If a two-winding transformer were

to perform the same duty, then weight of copper on its primary  $\alpha N_1 I_1$ ; weight of copper on secondary  $\alpha N_2 I_2$ . Total weight of copper  $\alpha (N_1 I_1 + N_2 I_2)$ 

 $\frac{\text{wieght of the copper in auto tarnsformer}}{\text{wieght of the copper in ordinary tarnsformer}} = \frac{(N_1 - N_2)I_1 + N_2(I_2 - I_1)}{N_1I_1 + N_2I_2}$ 

With simplification it is found that,

 $\frac{\text{wieght of the copper in auto tarnsformer}}{\text{wieght of the copper in ordinary tarnsformer}} = 1 - \frac{1}{a}$ 

Here we have assumed that  $N_1$  is greater than  $N_2$  i.e., *a* is greater than 1. The savings will of course be appreciable if the value of *a* is close to unity. For example if a = 1.2, copper required for autotransformer will be only 17% compared to a two winding transformer, i.e., saving will be about 83%. On the other hand, if a = 2, savings will be only 50%. Therefore, it is always economical to employ autotransformer where the voltage ratio change is close to unity. In fact autotransformers could be used with advantage, to connect two power systems of voltages say 11 kV and 15 kV.

#### **THREE PHASE TRANSFORMER**

generation, transmission and distribution of electrical power are more economical in three phase system than single phase system so the three phase system needs three single phase transformers. Three phase transformation can be done in two ways, by using single three phase transformer or by using a bank of three single phase transformers. Both are having some advantages over other. Single 3 phase transformer costs around 15 % less than bank of three single phase transformers. Again former occupies less space than later. The primary winding of each Phase is120° electrical degrees out of phase with the other two phases. In secondary winding each Phase is120° electrical degrees out of phase with the other two phases. Each primary winding is magnetically linked to one secondary winding through a common core leg. Sets of windings that are magnetically linked are drawn parallel to each other in the vector diagram. Transformer magnetizing currents are not purely sinusoidal, even if the exciting voltages are sinusoidal. The magnetizing currents have significant quantities of odd-harmonic components. The third, ninth, fifteenth and other so-called zero-sequence harmonic currents are in phase with each other; therefore, these components do not cancel out each other at the neutral but add in phase with one another to produce a zerosequence neutral current, provided there is a path for the neutral current to flow. Due to the nonlinear shape of the B-H curve, odd-harmonic magnetizing currents are required to support sinusoidal induced voltages. If some of the magnetizing current harmonics are not present, then the induced voltages cannot be sinusoidal. For very big transformer, it is impossible to transport large three phase transformer to the site and it is easier to transport three single phase transformers which is erected separately to form a three phase unit.

Another advantage of using bank of three single phase transformers is that, if one unit of the bank becomes out of order, then the bank can be run as open delta. The basic working principle of a three phase transformer is similar to the working principle of a single phase transformer. Power from primary is transferred to the secondary by the phenomenon of mutual induction. The main drawback in a three phase transformer is that, even if fault occurs in one phase, the whole transformer is removed from service for repairs.

# 6-1 Construction of Three Phase Transformer:

Three phase transformers can be of core type or shell type (just like single phase transformers). The constructional details of core type as well as shell type three phase transformers are as follows.

# 6-1-1 Core type construction:

The construction of a core type three phase transformer is as shown in the figure. The core consists of three legs or limbs. As usual, the core is made up of thin laminated sheets to reduce eddy current losses. Each limb has primary and secondary windings in cylindrical shape (former wound) arranged concentrically. The construction is well illustrated in the Fig.6-1.



Fig.6-1 Core type three phase transformer

# 6-1-2 Shell type construction:

In a shell type three phase transformer, three phases are more independent than they are in core type. Each phase has its individual magnetic circuit. The construction of shell type three phase transformer is illustrated in the Fig. 6-2. The construction is similar to that of three single phase shell type transformers kept on the top of each other.



Fig. 6-2 shell type three phase transformer

# **6-2 Three Phase Transformer Connections:**

A verity of connection of three phase transformer are possible on each side of both a single 3 phase transformer or a bank of three single phase transformers. The various configurations as The primary and secondary windings of a transformer can be connected in different configuration as shown in Fig. 6-3 to meet practically any requirement. In the case of three phase transformer windings, three forms of connection are possible: "star" (wye), "delta" (mesh) and "interconnected-star" (zig-zag). The combinations of the three windings may be with the primary delta-connected and the secondary star-connected, or star-delta, star-star or delta-delta, depending on the transformers use. When transformers are used to provide three or more phases they are generally referred to as a Poly phase Transformer.

### 6-2-1 Three phase transformer star and delta configurations:

But what do we mean by "star" (also known as Wye) and "delta" (also known as Mesh) when dealing with three-phase transformer connections. A three phase transformer has three sets of primary and secondary windings. Depending upon how these sets of windings are interconnected, determines whether the connection is a star or delta configuration. The three available voltages, which themselves are each displaced from the other by 120 electrical degrees, not only decided on the type of the electrical connections used on both the primary and secondary sides, but determine the flow of the transformers currents. With three single-phase transformers connected together, the magnetic flux's in the three transformers differ in phase by 120 time-degrees. With a single the three-phase transformer there are three magnetic flux's in the core differing in time-phase by 120 degrees. The standard method for marking three phase transformer windings is to label the three primary windings with capital (upper case) letters A, B and C, used to represent the three individual phases of the first phase A, the second phase B and the third phase C. The secondary windings are labelled with small (lower case) letters a, b and c. Each winding has two ends normally labelled 1 and 2 so that, for example, the second winding of the primary has ends which will be labelled B1 and B2, while the third winding of the secondary will be labelled c1 and c2 as shown in Fig. 6-4.



Fig. 6-3 Different forms for connections of the three phase transformer



Fig. 6-4 Star and delta configuration connections

The symbols are generally used on a three phase transformer to indicate the type or types of connections used with upper case Y for star connected, D for delta connected and Z for interconnected star primary windings, with lower case y, d and z for their respective secondaries. Then, Star-Star would be labelled Yy, Delta-Delta would be labelled Dd and interconnected star to interconnected star would be Zz for the same types of connected transformers. there are four different ways in which three single-phase transformers may be connected together between their primary and secondary three-phase circuits.

These four standard configurations are given as: Delta-Delta (Dd), Star-Star (Yy), Star-Delta (Yd), and Delta-Star (Dy). Transformers for high voltage operation with the star connections has the advantage of reducing the voltage on an individual transformer, reducing the number of turns required and an increase in the size of the conductors, making the coil windings easier and cheaper to insulate than delta transformers. The delta-delta connection nevertheless has one big advantage over the star-delta configuration, in that if one transformer of a group of three should become faulty or disabled, the two remaining ones will continue to deliver three-phase power with a capacity equal to approximately two thirds of the original output from the transformer unit. The following table shows the relation between for the line voltage and the phase voltage in case of star and delta connections also it shows the relation between for the line current and the phase current in case of star and delta connections.

connection	Line voltage	Phase voltage	Line current	Phase current
Star	$V_{\rm L} = \sqrt{3} \times V_{\rm P}$	$V_P = V_L \div \sqrt{3}$	$I_L = I_P$	$I_P = I_L$
Delta	$V_L = V_P$	$V_P = V_L$	$I_L = \sqrt{3} \times I_P$	$I_{\rm P} = I_{\rm L} \div \sqrt{3}$

Table 6-1 the relation between the line and phase for voltage and current

### <u>6-2-2 Star-star (Y-Y):</u>

In the star-star connection, each primary and secondary winding is connected to a neutral point. The neutral point may or may not be brought out to an external physical connection and the neutral may or may not be grounded. The connection of primary neutral to the neutral of generator has an add advantage that it eliminates distortion in the secondary phase voltages. If the flux in the core has sinusoidal waveform then it will give sinusoidal waveform for the

voltage. But due to characteristic of iron, a sinusoidal waveform of flux requires a third harmonic component in the exciting current. As the frequency of this component is thrice the frequency of circuit at any given constant. It will try to flow either towards or away from the neutral point in the transformer windings. With isolated neutral, the triple frequency current cannot flow so the flux in the core will not be a sine wave and the voltages are distorted. If primary neutral is connected to generator neutral the triple frequency currents get the path to solve the difficulty. The alternative way of overcoming with this difficulty is the use of tertiary winding of low KVA rating. These windings are connected in delta and provide a circuit in which triple frequency currents can flow. Thus sinusoidal voltage on primary will give sinusoidal voltage on secondary side. This situation changes if the neutrals of both sets of the primary and secondary windings are not grounded i.e. if the neutrals of both the primary and the secondary are open-circuited and so there is no path for the zero-sequence harmonic currents to flow and the induced voltages will not be sinusoidal. Analysis of the voltages induced by the "primary windings" is greatly complicated by the fact that the core is highly nonlinear so that each of the individual zero-sequence harmonics currents carried by the phantom primary windings will induce even higherorder harmonic voltages as well. Fourier analysis can be used to arrive at an approximation of the secondary voltages with an open primary neutral. Taking one phase at a time, the normal magnetizing current for a sinusoidal exciting voltage is plotted from the B-H curve of the transformer. The normal magnetizing current is converted to a Fourier series and then it is reconstructed by removing all of the zero-sequence harmonics. The resulting exciting current will have a shape different from the normal exciting current, which is then used to construct an induced voltage using the B-H curve in there verse manner that was used to construct the original exciting current. This process is rather laborious, so suffice it to say that if a Y-Y transformer does not have a neutral path for zero-sequence exciting currents, there will be harmonic voltages induced in the secondary even if the exciting voltage is purely sinusoidal. Star-star connection is generally used for small, high-voltage transformers. Because of star connection, number of required turns per phase is reduced (as phase voltage in star connection is  $\frac{1}{\sqrt{3}}$  times of line voltage

only) this means that, the number of turns in a transformer winding for star connection is 57.7 per cent, of that required for delta connection. Thus, the amount of insulation required is also reduced. The ratio of line voltages on the primary side and the secondary side is equal to the transformation ratio of the transformers. Line voltages on both sides are in phase with each other. This connection can be used only if the connected load is balanced. The star connection requires the use of three transformers, and if any one transformer becomes fault or disabled, the whole group might become disabled. Nevertheless, the star connected three phase transformer is especially convenient and economical in electrical power distributing systems, in that a fourth wire may be connected as a neutral point, (n) of the three star connected secondaries as shown in Fig. 6-5.



Fig. 6-5 Star star connections for three phase transformer

## 6-2-2-1 Advantages of Y-Y connection:

No phase displacement The primary and secondary circuits are in phase; i.e., there are no phase angle displacements introduced by the Y-Y connection. Required Few Turns for winding Due to star connection, phase voltages is  $\frac{1}{\sqrt{3}}$  times the line voltage. Hence less number of turns is required. Also the stress on insulation is less. This makes the connection economical for small high voltage purposes. Required Less Insulation Level If the neutral end of a Y-connected winding is grounded, then there is an opportunity to use reduced levels of insulation at the neutral end of the winding. A winding that is connected across the phases requires full insulation throughout the winding. Handle Heavy Load Due to star connection, phase current is same as line current. Hence windings have to carry high currents. This makes cross section of the windings high. Thus the windings are mechanically strong and windings can bear heavy loads and short circuit current.

# 6-2-2-2 Use for three phases four wires system:

Eliminate distortion in secondary phase voltage: The connection of primary neutral to the neutral of generator eliminates distortion in the secondary phase voltages by giving path to triple frequency currents toward to generator. Sinusoidal voltage on secondary side Neutral give path to flow Triple frequency current to flow Generator side thus sinusoidal voltage on primary will give sinusoidal voltage on secondary side. Used as Auto Transformer. A Y-Y transformer may be constructed as an autotransformer, with the possibility of great cost savings compared to the two-winding transformer construction. Better protective relaying The protective relay settings will be protecting better on the line to ground faults when the Y-Y transformer connections with solidly grounded neutrals are applied.

## 6-2-2-3 Disadvantages of Y-Y connection:

The Third harmonic issue: The voltages in any phase of a Y-Y transformer are 1200 apart from the voltages in any other phase. However, the thirdharmonic components of each phase will be in phase with each other. Nonlinearities in the transformer core always lead to generation of third harmonic. These components will add up resulting in large (can be even larger than the fundamental component) third harmonic component.

Over voltage at Lighting Load: The presence of third (and other zerosequence) harmonics at an ungrounded neutral can cause overvoltage conditions at light load. When constructing a Y-Y transformer using singlephase transformers connected in a bank, the measured line-to-neutral voltages are not 57.7% of the system phase-to-phase voltage at no load but are about 68% and diminish very rapidly as the bank is loaded. The effective values of voltages at different frequencies combine by taking the square root of the sum of the voltages squared. With sinusoidal phase-to-phase voltage, the thirdharmonic component of the phase-to-neutral voltage is about 60%.

Voltage drop at unbalance load: There can be a large voltage drop for unbalanced phase-to-neutral loads. This is caused by the fact that phase-tophase loads cause a voltage drop through the leakage reactance of the transformer whereas phase-to-neutral loads cause a voltage drop through the magnetizing reactance, which is 100 to 1000 times larger than the leakage reactance.

Overheated Transformer Tank: Under certain circumstances, a Y-Y connected three-phase trans- can produce severe tank overheating that can quickly destroy the transformer. This usually occurs with an open phase on the primary circuit and load on the secondary.

Over excitation of core in fault condition: If a phase-to-ground fault occurs on the primary circuit with the primary neutral grounded, then the phase-toneutral voltage on the un faulted phases increases to 173% of the normal voltage. This would almost certainly result in over excitation of the core, with greatly increased magnetizing currents and core losses. If the neutrals of the primary and secondary are both brought out, then a phase-to-ground fault on the secondary circuit causes neutral fault current to flow in the primary circuit. Ground protection re- laying in the neutral of the primary circuit may then operate for faults on the secondary circuit. Neutral shifting: If the load on the secondary side unbalanced then the performance of this connection is not satisfactory then the shifting of neutral point is possible. To prevent this, star point of the primary is required to be connected to the star point of the generator.

Distortion of secondary voltage: Even though the star or neutral point of the primary is earthed, the third harmonic present in the alternator voltage may appear on the secondary side. This causes distortion in the secondary phase voltages.

Over Voltage at Light Load: The presence of third (and other zero-sequence) harmonics at an ungrounded neutral can cause overvoltage conditions at light load.

Difficulty in coordination of ground Protection: In Y-Y Transformer, a lowside ground fault causes primary ground fault current, making coordination more difficult.

## 6-2-2-5 Top application:

This Type of Transformer is rarely used due to problems with unbalanced loads. It is economical for small high voltage transformers as the number of turns per phase and the amount of insulation required is less.

# <u>6-2-3 Star-delta (Y-Δ):</u>

The primary winding is star connected with grounded neutral and the secondary winding is delta connected. as shown in the Fig. 6-6. This connection is mainly used in step down transformer at the substation end of the transmission line Y connection on the HV side reduces insulation costs

the neutral point on the HV side can be grounded, stable with respect to unbalanced loads. As for example, at the end of a transmission line. The neutral of the primary winding is earthed. The ratio of secondary to primary line voltage is  $\frac{1}{\sqrt{3}}$  times the transformation ratio. There is 30° shift between

the primary and secondary line voltages. This type of connection has some advantages as, the primary side is star connected. Hence fewer numbers of turns are required. This makes the connection economical for large high voltage step down power transformers. The neutral available on the primary can be earthed to avoid distortion. The neutral point allows both types of loads (single phase or three phases) to be met. Large unbalanced loads can be handled satisfactory. The Y-D connection has no problem with third harmonic components due to circulating currents in D. It is also more stable to unbalanced loads since the D partially redistributes any imbalance that occurs. The delta connected winding carries third harmonic current due to which potential of neutral point is stabilized. Some saving in cost of insulation is achieved if HV side is star connected. But in practice the HV side is normally connected in delta so that the three phase loads like motors and single phase loads like lighting loads can be supplied by LV side using three phase four wire system. This connection has some disadvantages as, in this type of connection, the secondary voltage is not in phase with the primary. Hence it is not possible to operate this connection in parallel with star-star or deltadelta connected transformer. One problem associated with this connection is that the secondary voltage is shifted by  $30^{\circ}$  with respect to the primary voltage. This can cause problems when paralleling 3-phase transformers since transformers secondary voltages must be in-phase to be paralleled. Therefore, we must pay attention to these shifts. If secondary of this transformer should

be paralleled with secondary of another transformer without phase shift, there would be a problem.



Fig. 6-6 Star delta connections for three phase transformer

## <u>6-2-5 Delta star- (Δ-Y):</u>

The primary winding is connected in delta and the secondary winding is connected in star with neutral grounded as shown in Fig. 6-7. Thus it can be used to provide 3-phase 4-wire service. This type of connection is mainly used in step-up transformer at the beginning of transmission line. The ratio of secondary to primary line voltage is  $\sqrt{3}$  times the transformation ratio. There is 30° shift between the primary and secondary line voltages. It has some advantages as, Cross section area of winding is less at Primary side, used at Three phase four wire System, no distortion of Secondary Voltage due to third harmonic components, handled large unbalanced Load Large unbalanced loads can be handled without any difficulty. Grounding Isolation between Primary and Secondary Assuming that the neutral of the Y-connected secondary circuit is grounded, a load connected phase-to-neutral or a phaseto-ground fault produces two equal and opposite currents in two phases in the primary circuit without any neutral ground current in the primary circuit. Therefore, in contrast with the Y-Y connection, phase-to-ground faults or current unbalance in the secondary circuit will not affect ground protective relaying applied to the primary circuit. This feature enables proper coordination of protective devices and is a very important design consideration.



Fig. 6-7 Delta star connections for three phase transformer The neutral of the Y grounded is sometimes referred to as a grounding bank, because it provides a local source of ground current at the secondary that is isolated from the primary circuit. It has some advantages as, In the  $\Delta$  -Y connection, however, the third harmonic currents, being equal in amplitude and in phase with each other, are able to circulate around the path formed by the  $\Delta$  connected winding. The same thing is true for the other zerosequence harmonics. Protective relaying is much easier on a delta-wye transformer because ground faults on the secondary side are isolated from the primary, making coordination much easier. If there is upstream relaying on a delta-wye transformer, any zero-sequence current can be assumed to be from a primary ground fault, allowing very sensitive ground fault protection. It is commonly used in a step-up transformer, used universally for connecting generators

#### <u>6-2-4 Delta-delta ( $\Delta$ - $\Delta$ ):</u>

In a delta connected ( Dd ) group of transformers, the line voltage,  $V_L$  is equal to the supply voltage,  $V_L = V_S$ . But the current in each phase winding is given as:  $\frac{1}{\sqrt{3}}$  of the line current, where  $I_L$  is the line current. This connection is generally used for large, low-voltage transformers. Number of required phase per turns is relatively greater than that for star-star connection. The ratio of line voltages on the primary and the secondary side is equal to the transformation ratio of the transformers. This connection can be used even for unbalanced loading. This connection can be used even for unbalanced loading. One disadvantage of delta connected three phase transformers is that each transformer must be wound for the full-line voltage, and for 57.7 per cent, line current. The greater number of turns in the winding, together with the insulation between turns, necessitate a larger and more expensive coil than the star connection. Another disadvantage with delta connected three phase transformers is that there is no "neutral" or common connection. Delta delta connection of three phase transformer can be seen in Fig. 6-8.

## 6-2-5 Open delta connection:

some power system designers choose to create a three-phase transformer bank with only two transformers, representing a  $\Delta$ - $\Delta$  configuration with a missing winding in both the primary and secondary sides as shown in Fig. 6-9. This configuration is called "V" or "Open- $\Delta$ ." Of course, each of the two transformers have to be oversized to handle the same amount of power as three in a standard  $\Delta$  configuration, but the overall size, weight, and cost advantages are often worth it. Bear in mind, however, that with one winding set missing from the  $\Delta$  shape, this system no longer provides the fault tolerance of a normal  $\Delta$ - $\Delta$  system. If one of the two transformers were to fail, the load voltage and current would definitely be affected.



Fig. 6-8 Delta delta connections for three phase transformer



Fig. 6-9 Open delta connections
#### 5-2-7 The zigzag connection of transformer:

ZIg Zag transformer is a special purpose transformer used in power system, which is also called as "interconnected start winding". Even though this type of transformer connection is not useful in transforming power, it has many features combining Star and Delta type winding connections. Zig Zag transformer has six coils in which three are outer coils and three are inner coils as shown in the figure. The outer coil windings are called as ZIG winding and inner coil windings are connected as ZAG winding. The zig winding of one phase is connected in series with the zag winding of another phase so it is called interconnected star winding where two-star winding coils are interconnected each other as shown in Fig. 6-10. In each phase two coil windings will have same number of turns but they are wounded in opposite directions to cancel the mismatch voltages. Following are the connections of zig zag winding coils

1-The outer coil of phase 'a' is connected with the inner coil of phase 'b'
2- The outer coil of phase 'b' is connected with the inner coil of phase 'c'
3- The outer coil of phase 'c' is connected with the inner coil of phase 'c'
The inner coil second terminals are connected together and connected to neutral terminal to pass the zero sequence current components. The interconnection of winding of different phases introduces 30 phase shift between zig winding and the corresponding line to neutral voltage. The zig-zag winding have 15.47% more turns compared to conventional transformers to get the same magnitude of voltages. Hence the cost of zig zag transformer is high and it is essential in some applications to be used. Zig zag transformer

has some advantages as, provides low impedance to zero sequence currents and harmonic voltages suppression.



Fig. 6-10 Example for zigzag connection in the three phase transformer

### <u>6-2-7-1 Zig Zag Transformer applications or uses:</u>

Grounding transformer: It can be used as earthing transformer in a delta connected (no neutral terminal) system or an ungrounded start connected (three terminal star) where in neutral is not available for grounding. The zig zag transformer used for earthing of delta connected transformer. In delta connected transformer there will not be path to zero sequence components and no protection can be performed for these components which increases and stress and heating in the windings. The zig zag transformer provides a neutral for the proving a path to zero sequence components during line to ground fault and allows the protection to be operated due to this fault. In the absence of grounded neutral, voltages of healthy would increase line to line voltage level, stressing the insulation connected to equipment. Thug Zig zag transformer not only helps in protection it also reduces the voltages stress under symmetrical fault conditions. Power Electronic converters: In power electronic converters the zig zag transformer is used to eliminate the DC magnetizing component presented due to improper firing angles. The improper firing angles of power electronic components (SCR) may introduce DC magnetizing component and this is canceled in each limb of zig zag transformer due to opposite direction of DC magnetizing component of currents flowing in the windings on the same limb.

Earthing reference or earthing transformer: Zig zag transformer offers low impedance path to zero sequence components under fault conditions so it can be perfectly used as earthing transformer with and earthing reference. If the earthing current has to be limited under fault conditions, a suitable resistor can be placed in zig zag neutral terminal.

### 6-2-8 Scott (T-T) connection:

Scott (T-T) connection is a type of circuit used to derive two-phase electric power from a three-phase source, or vice versa Two transformers are used in this type of connection. One of the transformers has center taps on both primary and secondary windings (which is called as main transformer). The other transformer is called as teaser transformer. Scott connection can also be used for three phase to two phase conversion. The connection is made as shown in the Fig. 11.



Fig. 11 Scott (T-T) connection

# 6-3 Transformer Cooling:

There are several factors contributing to power losses in transformers. These are copper losses, which represent the major source of losses in a transformer, and core losses; namely hysteresis and eddy current losses. These losses components are produced in the form of heat energy which should be dissipated in a quick and rather efficient manner. If the transformer has otherwise failed to get rid of such heat generated, many problems could arise and in some cases severe consequences may occur. In fact, the improperly dissipated heat would further accumulate and thus cause the transformer temperature to increase. This process may lead to failure of paper insulation and liquid insulation medium of the transformer. Furthermore, excessive heat may result in damage of the transformer windings, the matter which, in particular, is considered as a catastrophe for expensive high power rating transformers. Therefore, numerous ways are introduced to keep the temperature within acceptable limits which in turn would help to maintain a long lifetime of the transformer. Various types of cooling arrangements have been used in transformers. These different cooling schemes are identified in electrical standards and are given the following naming convention.

6-3-1 For Dry-type transformers:

Self-air cooled (for transformers up to 3 MVA)

This method depends on the transformer surrounding air flow to naturally cool down the unit.

Forced Air Cooled (for transformers up to 15 MVA)

Air is pushed blowers to circulate through the transformer windings. This cause the air to heat up and then it starts to be cooled by ambient natural air.

6-3-2 For oil-Immersed transformers:

There are many types oil cooling such as oil natural air natural (ONAN), oil natural air forced (ONAF), oil forced air forced (OFAF), oil forced water forced (OFWF), oil directed air force (ODAF) and oil directed water forced (ODWF). In the following sections, the oil cooling systems are discussed

### 6-3-2-1 oil natural air natural (ONAN) transformer cooling:

This method stands for oil natural air natural is shown in Fig. 6-12. The cooling purpose is achieved by that the hot mineral oil is naturally circulated throughout loaded transformer through convection. In this simple cooling method, cold oil replaces the hot oil already flown to the top part of the transformer tank. This heat carried by the insulating medium is dissipated out to the ambient atmosphere through walls of transformer tank. All heat transfer mechanisms [i.e. conduction, convection, and radiation] are employed in this process. This implies the dissipation- process dependence on the tank surface area. That is why radiators or tubes/fins are usually used to effectively solve the tradeoff between the required increase in surface area available for heat transfer and the undesired increase in tank size. While these radiator banks are utilized for larger transformers, integrated fins are however used for smaller ones. Generally, this method of transformer cooling is used with low rating distribution-type transformers.



Fig. 6-12 Direction convection flow of oil transformer

# 6-3-2-2 oil natural air forced (ONAF) transformer cooling:

This method stands for oil natural air forced is shown in Fig. 6-13. For larger transformers units, electric fans can be installed as a mean for forced air cooling. This method depends in principal on blowing air on the cooling dedicated surface. This way it can thus accelerate the rate of dissipation of heat generated. This is due the increased amount of air pushed across the surface, which makes this cooling method offer improved cooling performance compared to ONAN method. The transformer loading capacity is therefore allowed to be increased by about 20-30 % without being at risk exceeding the temperature limits. It is worth noting in this arrangement that, similar to ONAN, natural convectional flow still govern the circulation of heated oil. This cooling method best suits large step up and step down outdoor transformers used in transmission power networks.



Fig. 6-13 Oil natural air forced cooling transformer

6-3-2-3 oil forced air forced (OFAF) transformer cooling:

This method stands for oil forced air forced. It had been found that the underlying principal of increasing the heat removal rate could be further improved when an accelerating force is applied to the circulation path of oil in the transformer tank. In this regard, oil pumps are used in large transformers where the radiators and fans do not suffice to meet their cooling requirements. In addition, from an economic perspective, it may be advantageous to benefit from this method's much lower space requirements when compared to those imposed by simple radiator batteries. These compact circulation pumps have their motors already submerged into oil. Noise levels produced are deemed to be very low in comparison to that produced by the transformer equipment itself.

#### <u>6-3-2-4 oil forced water forced (OFWF) transformer cooling:</u>

This method stands for oil forced water forced. Water can be considered a better substitute for air as a medium for heat exchange. For same climate conditions, this is due to water temperature is much lower than that of air. Although oil is forced toward the heat exchanger using pumps similar to OFAF method; however, this time the heat exchanger used is an oil to water one. The cooling process is achieved through cold water being sprayed over the hot oil flowing through the exchanger's piping system. Two coolers are used although only one is sufficient for the transformer operation; however, a tripping action should be activated if both fails. The water coolers are positively compact in size yet they can be negatively affected by the transformer environmental operating conditions. This is especially true in very low temperature surroundings where water may freezes and in very hot climate where abundance of water should be secured for proper operation. Moreover, the problem of water tube materials subject to corrosion may call for utilization of more complex materials like titanium during cooler manufacturing process. More importantly, ensuring totally sealed and isolated systems is crucial to avoid risky leakage into oil. This type of cooling is used for very large power transformers [above 500 MVA]



Fig. 6-14 Oil forced water forced cooling transformer 6-3-2-5 oil directed air force (ODAF) transformer cooling:

This method stands for oil directed air force. This transformer cooling scheme is the modified version of OFAF. To ensure higher rate of heat clearance, guided flowing paths between the insulated windings inside the transformer are provided for the oil coolant to pass through. This cooling method is specifically appropriate for transformers with considerably high power ratings.

## 6-3-2-6 oil directed water forced (ODWF) transformer cooling:

This method stands for Oil Directed Water Forced. This cooling system is similar to that discussed for ODAF except for the fluid used for cooling down the hot oil. Instead of air, water is forced in the cooler as a cooling system. If Pumps and fans suffer breakdown for some reason and need to be inspected, this should be done without affecting the service continuity of the transformer. Therefore various valves are used to make it possible, if necessary, to temporarily disconnect individual oil circuits for maintenance or replacement. In addition, a given transformer can have a combination of cooling types, for example ONAN/ONAF, to allow a change in the type of cooling.

## <u>6-4 parallel operation of three phase transformer:</u>

For supplying a load in excess of the rating of an existing transformer, two or more transformers may be connected in parallel with the existing transformer. The transformers are connected in parallel when load on one of the transformers is more than its capacity. The reliability is increased with parallel operation than to have single larger unit. The cost associated with maintaining the spares is less when two transformers are connected in parallel. It is usually economical to install another transformer in parallel instead of replacing the existing transformer by a single larger unit. The cost of a spare unit in the case of two parallel transformers (of equal rating) is also lower than that of a single large transformer. In addition, it is preferable to have a parallel transformer for the reason of reliability. With this at least half the load can be supplied with one transformer out of service.

### 6-4-1 Condition for parallel operation of transformer:

- For parallel connection of transformers, primary windings of the Transformers are connected to source bus-bars and secondary windings are connected to the load bus-bars.
- Various conditions that must be fulfilled for the successful parallel operation of transformers:
- 1. Same voltage Ratio & Turns Ratio (both primary and secondary Voltage Rating is same).
- 2. Same Percentage Impedance and X/R ratio.
- 3. Identical Position of Tap changer.

- 4. Same KVA ratings.
- 5. Same Phase angle shift (vector group are same).
- 6. Same Frequency rating.
- 7. Same Polarity.
- 8. Same Phase sequence.
- Some of these conditions are convenient and some are mandatory.
- The convenient are: Same voltage Ratio & turns ratio, same percentage impedance, same KVA rating, same position of tap changer.
- The mandatory conditions are: same phase angle shift, same polarity, same phase sequence and same frequency.
- When the convenient conditions are not met paralleled operation is possible but not optimal.

1.Same voltage Ratio & Turns Ratio (on each tap):

- If the transformers connected in parallel have slightly different voltage ratios, then due to the inequality of induced emfs in the secondary windings, a circulating current will flow in the loop formed by the secondary windings under the no-load condition, which may be much greater than the normal no-load current.
- The current will be quite high as the leakage impedance is low. When the secondary windings are loaded, this circulating current will tend to produce unequal loading on the two transformers, and it may not be possible to take the full load from this group of two parallel transformers (one of the transformers may get overloaded).

- If two transformers of different voltage ratio are connected in parallel with same primary supply voltage, there will be a difference in secondary voltages.
- Now when the secondary of these transformers are connected to same bus, there will be a circulating current between secondary's and therefore between primaries also. As the internal impedance of transformer is small, a small voltage difference may cause sufficiently high circulating current causing unnecessary extra I<sup>2</sup>R loss.
- The ratings of both primaries and secondary's should be identical. In other words, the transformers should have the same turn ratio i.e. transformation ratio.
- 2. Same percentage impedance and X/R ratio:
  - If two transformers connected in parallel with similar per-unit impedances they will mostly share the load in the ration of their KVA ratings. Here Load is mostly equal because it is possible to have two transformers with equal per-unit impedances but different X/R ratios. In this case the line current will be less than the sum of the transformer currents and the combined capacity will be reduced accordingly.
  - A difference in the ratio of the reactance value to resistance value of the per unit impedance results in a different phase angle of the currents carried by the two paralleled transformers; one transformer will be working with a higher power factor and the other with a lower power factor than that of the combined output. Hence, the real power will not be proportionally shared by the transformers.
  - The current shared by two transformers running in parallel should be proportional to their MVA ratings.

- The current carried by these transformers are inversely proportional to their internal impedance.
- From the above two statements it can be said that impedance of transformers running in parallel are inversely proportional to their MVA ratings. In other words percentage impedance or per unit values of impedance should be identical for all the transformers run in parallel.
- When connecting single-phase transformers in three-phase banks, proper impedance matching becomes even more critical. In addition to following the three rules for parallel operation, it is also a good practice to try to match the X/R ratios of the three series impedances to keep the three-phase output voltages balanced.
- When single-phase transformers with the same KVA ratings are connected in a Y-Δ Bank, impedance mismatches can cause a significant load unbalance among the transformers
- Lets examine following different type of case among Impedance, Ratio and KVA.
- If single-phase transformers are connected in a Y-Y bank with an isolated neutral, then the magnetizing impedance should also be equal on an ohmic basis. Otherwise, the transformer having the largest magnetizing impedance will have a highest percentage of exciting voltage, increasing the core losses of that transformer and possibly driving its core into saturation
- 3. Same polarity:
  - Polarity of transformer means the instantaneous direction of induced emf in secondary. If the instantaneous directions of induced secondary emf in two transformers are opposite to each other when same input

power is fed to the both of the transformers, the transformers are said to be in opposite polarity.

- The transformers should be properly connected with regard to their polarity. If they are connected with incorrect polarities then the two emfs, induced in the secondary windings which are in parallel, will act together in the local secondary circuit and produce a short circuit.
- Polarity of all transformers run in parallel should be same otherwise huge circulating current flows in the transformer but no load will be fed from these transformers.
- If the instantaneous directions of induced secondary emf in two transformers are same when same input power is fed to the both of the transformers, the transformers are said to be in same polarity.
- 4. Same phase sequence:
  - The phase sequence of line voltages of both the transformers must be identical for parallel operation of three-phase transformers. If the phase sequence is an incorrect, in every cycle each pair of phases will get short-circuited.
  - This condition must be strictly followed for parallel operation of transformers.

5. Same phase angle shift:(zero relative phase displacement between the secondary line voltages):

• The transformer windings can be connected in a variety of ways which produce different magnitudes and phase displacements of the secondary

voltage. All the transformer connections can be classified into distinct vector groups.

- Group 1: Zero displacement Dd0, Dz0) phase (Yy0, 2:180° Group phase displacement (Yy6, Dd6, Dz6) 3: -30° displacement Group phase (Yd1. Dy1, Yz1) Group 4: +30° phase displacement (Yd11, Dy11, Yz11)
- In order to have zero relative phase displacement of secondary side line voltages, the transformers belonging to the same group can be paralleled. For example, two transformers with Yd1 and Dy1 connections can be paralleled.
- The transformers of groups 1 and 2 can only be paralleled with transformers of their own group. However, the transformers of groups 3 and 4 can be paralleled by reversing the phase sequence of one of them. For example, a transformer with Yd1 1 connection (group 4) can be paralleled with that having Dy1 connection (group 3) by reversing the phase sequence of both primary and secondary terminals of the Dy1 transformer.
- We can only parallel Dy1 and Dy11 by crossing two incoming phases and the same two outgoing phases on one of the transformers, so if we have a DY11 transformer we can cross B&C phases on the primary and secondary to change the +30 degree phase shift into a -30 degree shift which will parallel with the Dy1, assuming all the other points above are satisfied.

# 6. Same KVA ratings:

- If two or more transformer is connected in parallel, then load sharing % between them is according to their rating. If all are of same rating, they will share equal loads
- Transformers of unequal kVA ratings will share a load practically (but not exactly) in proportion to their ratings, providing that the voltage ratios are identical and the percentage impedances (at their own kVA rating) are identical, or very nearly so in these cases a total of than 90% of the sum of the two ratings is normally available.
- It is recommended that transformers, the kVA ratings of which differ by more than 2:1, should not be operated permanently in parallel.
- Transformers having different kva ratings may operate in parallel, with load division such that each transformer carries its proportionate share of the total load To achieve accurate load division, it is necessary that the transformers be wound with the same turns ratio, and that the percent impedance of all transformers be equal, when each percentage is expressed on the kva base of its respective transformer. It is also necessary that the ratio of resistance to reactance in all transformers be equal. For satisfactory operation the circulating current for any combinations of ratios and impedances probably should not exceed ten percent of the full-load rated current of the smaller unit.
- 7. Identical tap changer and its operation:
  - The only important point to be remembered is the tap changing switches must be at same position for all the three transformers and should check and confirm that the secondary voltages are same. When the voltage tap

need change all three tap changing switches should be operated identical for all transformers. The OL settings of the SF6 also should be identical. If the substation is operating on full load condition, tripping of one transformer can cause cascade tripping of all three transformers.

- In transformers Output Voltage can be controlled either by Off Circuit Tap Changer (Manual tap changing) or By On – Load Tap Changer-OLTC (Automatic Changing).
- In the transformer with OLTC, it is a closed loop system, with following components:
- (1) AVR (Automatic Voltage Regulator- an electronic programmable device). With this AVR we can set the Output Voltage of the transformers. The Output Voltage of the transformer is fed into the AVR through the LT Panel. The AVR Compares the SET voltage & the Output Voltage and gives the error signals, if any, to the OLTC through the RTCC Panel for tap changing. This AVR is mounted in the RTCC.
- (2) RTCC (Remote Tap Changing Cubicle): This is a panel consisting of the AVR, Display for Tap Position, Voltage, and LEDs for Raise & Lower of Taps relays, Selector Switches for Auto Manual Selection...
   In AUTO MODE the voltage is controlled by the AVR. In manual Mode the operator can Increase / decrease the voltage by changing the Taps manually through the Push Button in the RTCC.
- (3) OLTC is mounted on the transformer. It consists of a motor, controlled by the RTCC, which changes the Taps in the transformers.
- Both the Transformers should have same voltage ratio at all the taps & when you run transformers in parallel, it should operate as same tap position. If we have OLTC with RTCC panel, one RTCC should work

as master & other should work as follower to maintain same tap positions of Transformer.

• However, a circulating current can be flown between the two tanks if the impedances of the two transformers are different or if the taps of the on-load tap changer (OLTC) are mismatched temporarily due to the mechanical delay. The circulating current may cause the malfunction of protection relays.

Other necessary condition for parallel operation

- 1. All parallel units must be supplied from the same network.
- 2. Secondary cabling from the transformers to the point of paralling has approximately equal length and characteristics.
- 3. Voltage difference between corresponding phase must not exceed 0.4%
- 4. When the transformers are operated in parallel, the fault current would be very high on the secondary side. Supposing percentage impedance of one transformer is say 6.25 %, the short circuit MVA would be 25.6 MVA and short circuit current would be 35 kA.
- 5. If the transformers are of same rating and same percentage impedance, then the downstream short circuit current would be 3 times (since 3 transformers are in Parallel) approximately 105 kA. This means all the devices like ACBs, MCCBs, switch boards should withstand the short-circuit current of 105 kA. This is the maximum current. This current will get reduced depending on the location of the switch boards, cables and cable length etc. However this aspect has to be taken into consideration.
- 6. There should be Directional relays on the secondary side of the transformers.

 The percent impedance of one transformer must be between 92.5% and 107.5% of the other. Otherwise, circulating currents between the two transformers would be excessive.

## Problem 6-1:

A 3-phase, 500 kVA, 6000V/400V, 50Hz, delta-star connected transformer is delivering 300 kW, at 0.8 pf lagging to a balanced 3-phase load connected to the LV side with HV side supplied from 6000 V, 3- phase supply. Calculate the line and winding currents in both the sides. Assume the transformer to be ideal.

### Solution:

First note that it is not a bank of single phase transformers. In fact it is a single unit of 3-phase transformer with the name plate rating as 500 kVA, 6000 V/400 V, 50Hz, delta-star connected 3-phase transformer. 500 kVA represents the *total* kVA and voltages specified are always line to line. Similarly unless otherwise specified, kW rating of a 3-phase load is the total kW absorbed by the load. The connection diagram is shown in Fig. 6-15.



Fig. 6-15 Connection diagram with 3-phase load.

Noting the relation kVA,  $S = \frac{P}{\cos \phi_2}$  and  $I = \frac{S}{\sqrt{3}V_{L-L}}$  let us start out calculation.

Load kVA =  $\frac{300}{0.8}$  = 375*KV* A = input kVA

Line current drawn by the load,  $I_{L2} = \frac{375000}{\sqrt{3}x \, 400} = 541.3 \text{A}$ 

Because of star connection, LV coil current = 541.3 A

since input kVA = 375 kVA

HV side line current,  $I_{L1} = \frac{375000}{6000x \sqrt{3}} = 36.1 \text{A}$ 

Actual phase winding currents can also be calculated as:

LV side phase coil current = LV side line current

$$I_{ph2} = I_{L2} = 541.3$$
A

HV side phase coil current = LV side  $\frac{\text{line current}}{\sqrt{3}} = \frac{36.1}{\sqrt{3}} = 20.8\text{A}$ 

## Problem 6-2:

A 2000-kVA, 6,600/400-V, 3-phase transformer is delta-connected on the high voltage side and star-connected on the low-voltage side. Determine its % resistance and % reactance drops, % efficiency and % regulation on full load 0.8 p.f. leading given the following data :

S.C. test ; H.V. data : 400 V, 175 A and 17 kW

O.C. test; L.V. data : 400 V, 150 A and 15 kW

#### Solution:

From S.C. test data, we have Primary voltage/phase = 400 V; Primary current/phase =  $\frac{175}{\sqrt{3}}$  = 101A

$$Z_{01} = \frac{400}{101} = 3.96\Omega$$
$$R_{01} = \frac{P_{SC}}{3I^2} = \frac{17000}{3(101)^2} = 0.555\Omega$$

$$X_{01} = \sqrt{(3.96)^2 - (0.555)^2} = 3.92\Omega$$
  
% $R_{01} = \frac{I_1 R_{01}}{V_1} x 100 = \frac{101x \ 0.555}{6600} x 100 = 0.849$   
% $X_{01} = \frac{I_1 X_{01}}{V_1} x 100 = \frac{101x \ 3.92}{6600} x 100 = 6$ 

% regulation =  $v_r \cos \phi_2 - v_x \sin \phi_2 = 0.849 \times 0.8 - 6 \times 0.6 = 2.92\%$ 

It shows that S.C. test has been carried out under full-load conditions. Total losses= 17+15=32 kw

Output power =  $200 \times 0.8 = 160 \text{ kw}$ 

$$\eta = \frac{160}{160 + 32} \times 100 = 98\%$$

Problem 6-3:

A 5,000-kVA, 3-phase transformer, 6.6/33-kV,  $\Delta$ /Y, has a no-load loss of 15 kW and a full-load loss of 50 kW. The impedance drop at full-load is 7%. Calculate the primary voltage when a load of 3,200 kW at 0.8 p.f. is delivered at 33 kV.

Solution:

$$I_{L2} = \frac{5000000}{33000x \sqrt{3}} = 87.5 \text{A}$$

Impedance drop per phase =  $7\%(\frac{33000}{\sqrt{3}}) = 1333.679$  V

$$Z_{02} = \frac{1333.679}{87.5} = 15.3\Omega$$
 per phase

Full load copper loss = 50-15 = 35 kw

$$R_{02} = \frac{35000}{3x \, 87.5^2} = 1.53\Omega \text{ per phase}$$
$$X_{01} = \sqrt{(15.3)^2 - (1.53)^2} = 15.23\Omega$$

When load is 3,200 kW at 0.8 p.f.

$$I_{L2} = \frac{3200000}{33000x\sqrt{3}} = 70$$
A

The voltage drop =  $70(1.53x \ 0.8 + 15.23x \ 0.6) = 725v$  per phase

$$\% \operatorname{reg} = \frac{725 \times 100}{19000} = 3.8\%$$

Primary voltage will have to be increased by 3.8%.

: Primary voltage = 
$$6.6 + 6.6x \frac{3.8}{100} = 6.85 \text{ KV}$$